Engage
Education
Foundation

Units 3 and 4 Maths Methods (CAS): Exam 2

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Section A – Multiple-choice questions

Question 1
The correct answer is D.

Dice rolls are independent events. The chance of getting any number on a standard fair die is 1/6.

Question 2
The correct answer is D.

D is an unusual hybrid function, but it still passes the horizontal line test. Inverse functions for quadratic, quartic and periodic functions do not exit unless the domain is restricted to make them one-to-one functions.

Question 3
The correct answer is A.

Solving $f(x) = 6$ and $f(x) = 12$ give $x = 17 \text{ and } x = 65$, respectively. The domain must then include all real numbers between 17 and 65.

Question 4
The correct answer is B.

Due to the independence of the events A and B, we are given that $\Pr(A) * \Pr(B) = (1 - \Pr(A))(1 - \Pr(B))$. This rearranges to $\Pr(A) + \Pr(B) = 1$.

Question 5
The correct answer is A.

The z-value for 99% confidence interval is 2.58.

Question 6
The correct answer is B.

$\ln(x^2) + 2$ can also be written as $2\ln(x) + 2$. This question can easily be done by eliminating the other options, as they can easily be compared to $2\ln(x) + 2$, and seen to be different.

Question 7
The correct answer is E.

$ad = bc$ would give either no solution, or infinitely many solutions, and thus cannot be true. A, B, and D can be quickly seen to be true, while C may take a while to confirm.

Question 8
The correct answer is A.

This follows from an application of the chain rule to the given function.

Question 9
The correct answer is B.

The average value of a function across a range is given by its area divided by length. This particular function has 1 unit area, because it is a continuous probability distribution. Since it is defined for the continuous 2 unit length, its average value is $\frac{1}{2}$.

Question 10
The correct answer is C.

If we let the events described in C be A and B, then $\Pr(A \cap B) = 0$, and $\Pr(A) \times \Pr(B) = 0 \times 0 = 0 = \Pr(A \cap B)$, thus A and B are independent.

It can be verified that the other events are not independent. All of the choices except for D are mutually exclusive, not to be confused with independence.

Question 11
The correct answer is D.

$$\Pr(X > 18) = \Pr\left(Z > \frac{x - \mu}{\sigma}\right) = \Pr\left(Z > \frac{18 - (-3)}{15}\right)$$

$$= \Pr\left(Z > \frac{21}{15}\right) = \Pr\left(Z > \frac{7}{5}\right) = 1 - \Pr\left(Z < \frac{7}{5}\right) = 1 - \Pr(Z > -\frac{7}{5})$$

Note that answers do not need to be expressed in their most "obvious" forms. It is also a good idea to work backwards from the answers.

Question 12
The correct answer is E.

If $\frac{5}{2}$ is included in the domain, then setting $x = \frac{5}{2}$ results in division by zero. Therefore, $\frac{5}{2}$ cannot be in the domain, thus the answer must be E.

Question 13
The correct answer is C.

$\hat{p}$ is the average of 55 and 65%, hence $\hat{p} = 0.6$. The z-score for 90% confidence interval is 1.64. Since the lower limit of the confidence interval is 55%, we can use the left arm of the confidence interval formula $0.55 = \hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.6 - 0.64\sqrt{\frac{0.6*0.4}{n}}$. Therefore, solving for n gives n=40.

Question 14
The correct answer is E.

The given graph is a quadratic with lowest value at a negative x coordinate. Integrating the function $f'(x)$ would give a graph similar to C. Hence the function we require $f(-x)$ is a graph of C, reflected in the y-axis. The only such function is E.

Question 15
The correct answer is D.

If we let the side length be s, and the volume be V, then $V = s^3$ and $\frac{dV}{ds} = 3s^2$.

Hence, $\frac{ds}{dt} = \frac{ds}{dV} * \frac{dV}{dt} = \frac{72}{3s^2} = \frac{72}{30000} = \frac{3}{1250}$ mm/day

Question 16
The correct answer is A.

This question tests application of rules of addition and multiplication by constants in integration. The given equation simplifies to $-2\int_1^0 f(x)dx + 3 = 7$, which gives $\int_1^0 f(x)dx = -2$

Question 17
The correct answer is D.

Both D and E satisfy the given equation, but D is of less magnitude.

Question 18
The correct answer is D.

Only C and D satisfy $3f(0) = g(0) = 3$, but C does not satisfy $f'(x) = 2g'(x)$.

Question 19
The correct answer is E.

E is a standard parabola with turning point above the x-axis, and thus lacks an x-intercept.

Question 20
The correct answer is C.

Since the sum of the bottom row of the table must be 1, $c = \frac{1}{2} - a$. The question is then a simultaneous equation which solves to give $a = \frac{1}{3}$.

Question 21
The correct answer is D.

The image of a function $y = f(x)$ is $y = f(x) + \frac{7}{2}$ when the series of transformation is applied. It is also possible to answer this question by substituting a single point into the series of transformations. For instance, the point $(0,1)$ is sent to $(0,4.5)$, implying a translation of 3.5 units in the positive y direction.

Question 22
The correct answer is D.

For a polynomial, the number of roots of the derivative is the number of turning points of the original function. In this case, the original function has 3 turning points, hence the derivative has 3 roots.

Section B – Short-answer questions

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a i

The probability of buying a car in the next year is $\frac{1}{3}$, while it is $\frac{1}{5}$ for each of the next three years [1].

Thus, the probability of buying cars for the next four years is $\frac{1}{3} \times (\frac{1}{5})^3 = \frac{1}{375}$ [1]

Question 1a ii

The three ways to accomplish this are Car/Bike/Bike, Bike/Car/Bike, and Bike/Bike/Car [1]

Their probabilities are, respectively, $\frac{1}{3} \times \frac{4}{5} \times \frac{2}{3} = \frac{8}{45}$, $\frac{2}{3} \times \frac{1}{3} \times \frac{4}{5} = \frac{8}{45}$, and $\frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$ [1]

Hence, $\frac{8}{45} + \frac{8}{45} + \frac{4}{27} = \frac{68}{135}$ [1]

Question 1a iii

$\Pr(0 \text{ or } 2 \text{ or } 3 \text{ cars}) = 1 - \Pr(2 \text{ cars}) = 1 - \frac{68}{135} = \frac{67}{135}$ [1]

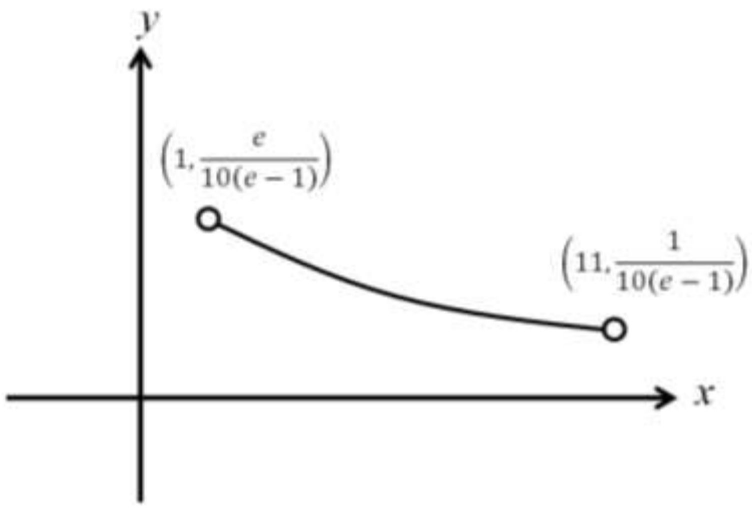
Question 1b

$\frac{\frac{1}{3}}{\frac{1}{3}+\frac{4}{5}} = \frac{5}{17} = 0.29$ [1]

This can also be done by calculator estimation.

Question 1c

The required graph is a downwards sloping curve from $\left(1, \frac{e}{10(e-1)}\right)$ to $\left(11, \frac{1}{10(e-1)}\right)$. In particular, it should be made clear that the endpoints of the curve are not part of the graph. Instead, $(1,0)$ and $(11,0)$ are open end points.



Correct shape [1]

Correct treatment of endpoints [1]

Question 1d

This probability is equal to the area of the probability density function between $x = 10$ and $x = 11$, which is:

$\int_{10}^{11} \frac{e^{\frac{11-x}{10}}}{10(e-1)}\, dx = 0.061$ [2]

Question 1e

The median price, m, in tens of thousands of dollars, of cars that Steve buys is given by solving the equation $\int_{m}^{11} \frac{e^{\frac{11-x}{10}}}{10(e-1)}\, dx = 0.5$ for m [1]

This gives $m = 4.80$

Hence the median price of cars is $\$4.80 \times 10^4$ [1]

Question 1f

This is given by evaluating $\int_{4.8}^{10} \frac{e^{\frac{11-x}{10}}}{10(e-1)}\, dx = 0.439$ [1]

Question 1g

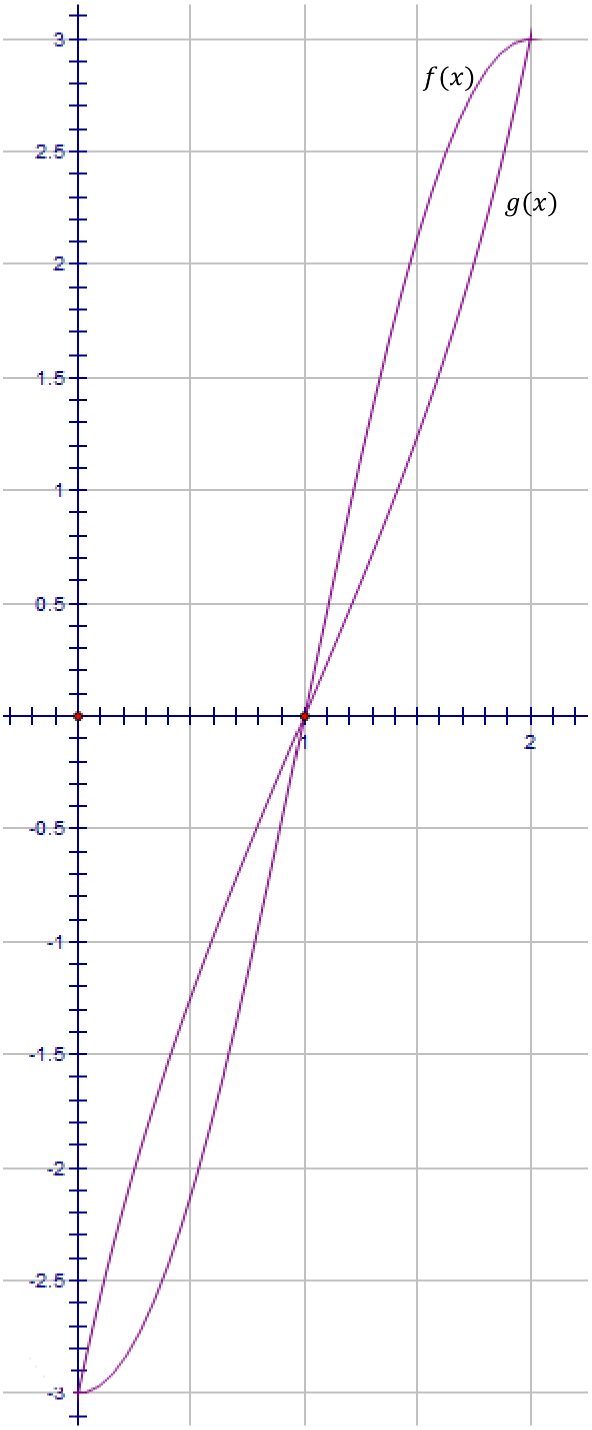
The mean price of cars is given by $\int_{1}^{11} xf(x)\, dx$= $\int_{1}^{11} x\left(\frac{e^{\frac{11-x}{10}}}{10(e-1)}\right) dx = 5.18$ (in tens of thousands of dollars) [1]

The probability of the car being more expensive than the mean is $\int_{5.18}^{11} \frac{e^{\frac{11-x}{10}}}{10(e-1)}\, dx = 0.460$ [1]

The chance that Steve decides to purchase a car above the mean price in a given year is $0.460 \times \frac{1}{4} = 0.115$ [1]

Thus, the chance of purchasing a car above the mean price two years in a row is $0.115^2 = 0.0132$ [1]

Question 2a



Correct shape [1] and labelling of graphs [1]

x and y-intercepts are $(0,-3),(1,0),(2,3)$ [1]

Question 2b

The derivative of $g(x)$ is $g'(x) = \frac{3\pi}{4}(\sec\frac{(x-5)\pi}{4})^2$ [1]

Graph and use the CAS calculator to find the maximum points, $(0, 4.71)$, and $(2, 4.71)$ [1]

Question 2c
The period of h is unaffected by multiplication by a constant. Hence, the period of $2h(x)$ is the same as that of $h(x)$ [1].

The standard period of the sine function can be found by dividing 2π by the coefficient of x to find that the period of h is 4. It can also be seen from the graph of $f(x)$ that half a period has been drawn, hence the period is 4 [1].

Question 2d i
There are numerous ways to do this. All of the following are correct. No working is required [2].

$$2\int_0^1 \big(g(x) - f(x)\big)dx$$

$$\int_0^1 \big(g(x) - f(x)\big)dx + \int_1^2 \big(f(x) - g(x)\big)dx$$

$$\int_0^2 |g(x) - f(x)|dx$$

Question 2d ii
Evaluating any correct integral gives an area of 1.17 [2]

Question 2e
The range of the inverse is equal to the domain of the original function, and the domain of the inverse is equal to the range of the original function. Thus, for both $f^{-1}(x)$ and $g^{-1}(x)$, the domain is $[-3,3]$, and the range is $[0,2]$ [1 mark for each function]

Question 2f
This is the same as the area found in question 2d ii, because the inverses are simply reflections of the original functions.

Thus, the required area is 1.17 [2]

Question 3a
$a + b + a + a + \frac{2}{5} + a = 1$ [1]

$4a + b = \frac{3}{5}, 8a + 2b = \frac{6}{5}$ [1]

Question 3b
$a + 2b + 3a + 4a + 2 + 6a = 4, 14a + 2b = 2$ [1]

Subtracting $8a + 2b = \frac{6}{5}$ gives $6a = \frac{4}{5}, a = \frac{2}{15}, b = \frac{1}{15}$ [1]

Question 3c
$$\frac{2}{15} + \frac{1}{15} + \frac{2}{15} + \frac{2}{15} < \frac{1}{2}, \text{but } \frac{2}{15} + \frac{1}{15} + \frac{2}{15} + \frac{2}{15} + \frac{2}{5} > \frac{1}{2}$$

Hence the median roll is 5 [1]

Question 3d
This equals $\Pr(X > 6) = \Pr\left(Z > \frac{2}{3}\right) = 0.252$ [1]

Question 3e
This equals,

$\frac{2}{15} * \Pr(X < 1) + \frac{1}{15} * \Pr(X < 2) + \frac{2}{15} * \Pr(X < 3) + \frac{2}{15} * \Pr(X < 4) + \frac{2}{5} * \Pr(X < 5) + \frac{2}{15} * \Pr(X < 6)$ [2]

Evaluating this expression using CAS calculator gives the answer 0.506 [2]

Question 3f
The chance of Bob winning a game is $1 - 0.506 = 0.494$ [1].

The chance of Bob winning three or more games can then be evaluated using the Binomial evaluative ability of the CAS calculator, or alternatively by calculating

$4 \times 0.494^3 \times 0.506 + 1 \times 0.494^4 = 0.304$ [2]

Question 4a i
$f(1) = 2$ gives $a + b + c = 2$ [1]

$f'(1) = 5$ gives $3a + 2b = 5$ [1]

$f'(-1) = 0$ gives $3a - 2b = 0$ [1]

Question 4aii
$a = \frac{5}{6}, b = \frac{5}{4}, c = -\frac{1}{12}$ [2]

[1] if one or two of the values are correct.

This can be done by calculator, or by hand. In this case, it is likely to be faster to solve by hand.

Question 4b
d and e are the first two solutions to $\frac{5x^3}{6} + \frac{5x^2}{4} - \frac{1}{12} = 0$

This should be done by CAS, resulting in the solution $d = -1.45, e = -0.29$ [2]

Question 4c
Evaluating $\int_{-1.45}^{-0.29} \frac{5x^3}{6} + \frac{5x^2}{4} - \frac{1}{12}\,dx$ gives 0.244 [2]

Question 4d
Solving $\int_{-1.45}^{-0.29} k\,(x - (-1.45))(x - (-0.29))dx = 0.244$ for k yields $k = -0.938$ [1]

Question 4e
$g(x) = -0.938(x + 1.45)(x + 0.29)$

$g'(x) = -1.876x - 1.632$ [1]

$f'(x) = \frac{5}{2}x^2 + \frac{5}{2}x$ [1]

The maximum of $g'(x)$ in the domain is 1.0882

The maximum of $f'(x)$ in the domain is 1.6312

The latter is larger; thus Cathy is right [1].