Engage
Education
Foundation

Units 3 and 4 Maths Methods (CAS): Exam 1

Practice Exam Solutions

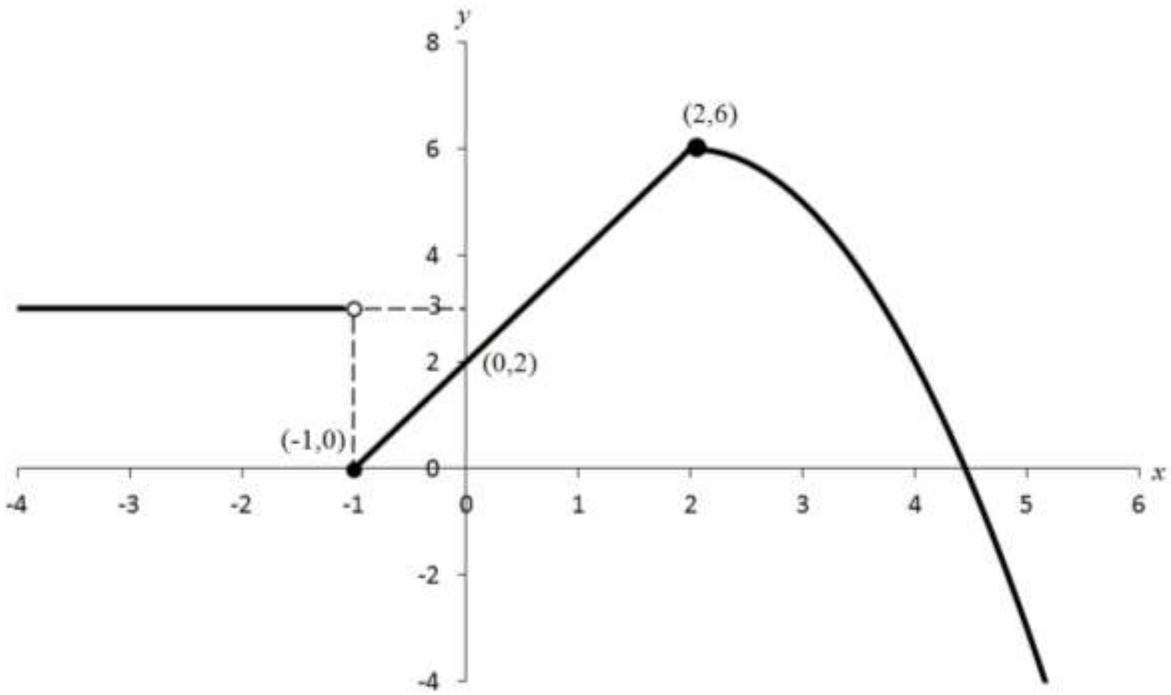
Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Question 1a



[2] for correct shape, showing correct included/excluded points, coordinates of key points.

Question 1b
$x = -1$ [1]

Question 1c
Range = $(-\infty, 6]$ [1]

Question 2a
Rearranging the given equation gives $\boldsymbol{4e^{2x}} - \boldsymbol{5e^{x}} + 1 = 0$

Solving the equation using quadratics yields $4e^x - 1 = 0 \; or \; e^x - 1 = 0$ [1]

Hence, $x = -2log_e(2) or \; 0$ [1]

Question 2b

If we let θ=3x, we get $\tan(\theta) = \frac{1}{\sqrt{3}}$ and θ∈ $(-\frac{3\pi}{2}, \frac{3\pi}{2})$

$\tan(\theta) = \frac{1}{\sqrt{3}}$ has solutions $\theta = -\frac{5\pi}{6}, \frac{\pi}{6}, \frac{7\pi}{6}$

[1] for correct application of domain

Therefore, the solutions to $\tan(3x) = \frac{1}{\sqrt{3}}$ must be $x = -\frac{5\pi}{18}, \frac{\pi}{18}, \frac{7\pi}{18}$ where $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ [1]

Question 3a
The range of f is $(-\infty, 3]$. So the domain of f^{-1} must also be $(-\infty, 3]$ [1]

For the rule, the inverse must satisfy $x = 3 - \log_e(y + 1)$, which rearranges to $y = e^{3-x} - 1$. Thus, we have $f^{-1}: (-\infty, 3] \to \mathbb{R}, f^{-1}(x) = e^{3-x} - 1$ [1]

Question 3b
Applying f to both sides gives $x = f(4)$. Hence, $x = 3 - \log_e(5)$

Or, since $f^{-1}(x) = e^{3-x} - 1 = 4$, we get $x = 3 - \log_e(5)$ [1]

Question 3c
$\mathrm{f(x)} = 6 - 2\log_e(\mathrm{x} - 1)$ [1]

Question 4a
Application of product rule gives $\frac{dy}{dx} = -(2x-1)^3 - 6x(2x-1)^2 = -(2x-1)^2(8x-1)$ [1]

Question 4b
From $f'(x) = (x-3)\sin(x) - \cos(x)$ using the chain rule, [1]
we find that $f'\left(\frac{\pi}{3}\right) = \frac{\pi\sqrt{3}}{6} - \frac{3\sqrt{3}}{2} - \frac{1}{2}$ [1]

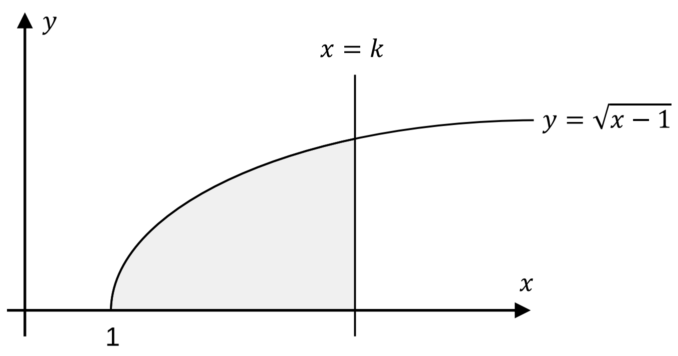
Question 4c
Since $\frac{dy}{dx} = 4x^3 + 20x = 4x(x^2+5)$, the stationary points where $\frac{dy}{dx} = 0$ occurs only when $x = 0$ [1]

A quick check shows:

x	-1	0	1
$\frac{dy}{dx}$	\	–	/

Hence, (0, -25) is a local minimum and the only stationary point of y [1]

Question 5
The area bound by three equations given is shown below:



Area $= \int_1^k \sqrt{x-1}\,dx$ [1]

$= \left[\frac{2}{3}(x-1)^{\frac{3}{2}}\right]_1^k$

$= \frac{2}{3}(k-1)^{\frac{3}{2}} - \frac{2}{3}(1-1)^{\frac{3}{2}}$

$= \frac{2}{3}(k-1)^{\frac{3}{2}} = 18$ [2]

Hence, we need to solve:

$\frac{2}{3}(k-1)^{\frac{3}{2}} = 18$
$(k-1)^{\frac{3}{2}} = 27$
$k - 1 = 9$
$k = 10$ [1]

Question 6a

The general equation of the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

Since the height of the conical pile is 6 times the radius ($h = 6r$), we can eliminate the variable h to yield $V = 2\pi r^3$ [1]

Differentiating V with respect to r gives $\frac{dV}{dr} = 6\pi r^2$ [1]

Question 6b
The rate of increase of the volume of the file is $\frac{dV}{dt} = 4\pi$. Hence, $V = 4\pi t$ [1]

At $t = 4$, $V = 16\pi = 2\pi r^3$. Therefore, $r = 2\ cm$ [1]

Question 6c
Using the chain rule, we get $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = \frac{1}{6\pi r^2} \times 4\pi = \frac{2}{3r^2}$ [1]

When $\mathrm{t} = 32$:

$\mathrm{V} = 4\pi \times 32 = 128\pi$

Therefore, $V = 2\pi r^3 = 128\pi$ and solving this equation gives $\mathrm{r} = 4$. Substituting $\mathrm{r} = 4$ into the rule of $\frac{dr}{dt}$ gives, $\frac{dr}{dt} = \frac{2}{3r^2} = \frac{2}{3\times 4^2} = \frac{1}{24} cm/sec$ [1]

Question 7a
$x = 1$ [1]

Question 7b
$\mu = 0 \times 0.2 + 1 \times 0.4 + 2 \times 0.25 + 3 \times 0.1 + 4 \times 0.05 = 1.4$ [1]

Question 7c
$0.2 \times 0.2 = 0.04$ [1]

Question 7d
We seek $\Pr(X = 2|X > 0)$. This equals $\frac{0.25}{0.8} = \frac{5}{16}$ [1]

Question 8a
$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.2*0.8}{40}} = \sqrt{\frac{1}{250}} = \frac{\sqrt{10}}{50}$

[1] for correct standard deviation formula

[1] mark for answer

Question 8b

$$\left(\hat{p} - \mathrm{z}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + \mathrm{z}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = \left(0.4 - 1.96\sqrt{\frac{0.4 * 0.6}{30}}, 0.4 + 1.96\sqrt{\frac{0.4 * 0.6}{30}}\right)$$

[2] for correct expression

Question 9a

In order to be a probability distribution, we need $\int_0^2 k\cos(kx)\,dx = 1$ [1]

Integration then yields $[\sin(kx)]_0^2 = \sin(2k) = 1$

Thus, $k = \frac{\pi}{4}$ since $0 \leq k < \pi$ [1]

Question 9b

$\Pr\left(X \leq \frac{2}{3} \middle| X \leq 1\right) = \frac{\left[\sin\left(\frac{\pi x}{4}\right)\right]_0^{\frac{2}{3}}}{\left[\sin\left(\frac{\pi x}{4}\right)\right]_0^{1}}$ [1 mark for conditional probability, 1 mark for integration]

$$= \frac{sin\left(\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{4}\right)}$$

$$= \frac{1/2}{1/\sqrt{2}}$$

$= \frac{\sqrt{2}}{2}$ [1]