E

Free Exam
for 20xx-xx VCE study design

Engage
Education
Foundation

Units 3 and 4 Further Maths: Exam 2

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Section A – Core

Data Analysis

Question 1a i
3.8kg

This is done by reading the dot plot from bottom to top, and finding the weight between the 15th and 16th dot [1].

Question 1a ii
20%

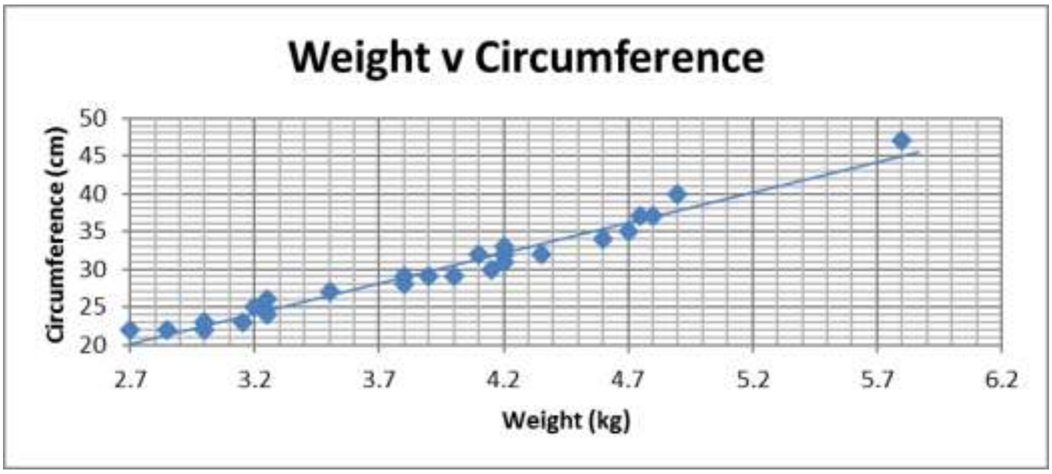
This is done by locating the line of 4.5kg on the dot plot, counting the number of dots about that line (6), and dividing that by the number of pumpkins in total [1].

$\frac{30}{6} \times 100 = 20\%$

Question 1b
2.5% [1]

Using the 68-95-99.7% rule, pumpkins with a circumference smaller than 16cm will fall 2 standard deviations below the mean (28-6-6=16). Therefore, only 5% of these pumpkins will have a circumference either smaller or greater than 2 standard deviations from the mean. Half of them will have the circumference smaller than the 2 standard deviations = $\frac{5}{2} = 2.5\%$

Question 2a



Question 2b
When the weight of a pumpkin is 0kg, the circumference of the pumpkin will be -1.5cm. As you can't have a 0kg pumpkin (because that means there's no pumpkin) and you can't have negative measurements, the values are not feasible in real life.

Question 2c
For every kilogram of pumpkin, the circumference will increase 8 times that.

Question 2d
Using the r^2 value, 98% of the variation in circumference can be explained by the variation in weight.

Question 2e
Circumference = -1.5 + 8.01 x 4.8 = 36.95 [1].

Residual = actual y value – predicted y value

Residual = 41 – 36.95 = 4.05 = 4cm [1].

Question 3a i
Rain – as the lower boarder of the rainy day box plot is 16, whilst the lower boarder of the sunny day box plot is 17 [1].

Question 3a ii
Sun – from the box plot, the range of sunny days is 28-17=11, whilst the range of rainy days is 23-16 = 7 [1].

Question 3b
23 – the median is the middle line on the box plot, which lines up with 23 [1].

Question 3c
4 – $IQR\ is\ Q3 - Q1 = 25 - 21 = 4$ [1].

Question 3d
[2] - 16, 17, 18, 19, 20, 21, 22, 23

Question 4a
23 degrees [1]

$t3pm = 2.02 + 0.83 \times 25 = 22.76 = 23$

Question 4b
Y^2 transformation therefore apply the transformation to the variable t3pm [1].

Question 5a
[2] for:

t8am	Stem	t3pm
	0	7
6, 4, 3, 2, 0 8, 8, 6	1	2, 3, 4, 7, 7, 7, 8, 8
3, 3, 0	2	0, 1

Question 5b
17 [1]

The mode is the most common number in the data set.

Question 5c
$\frac{10+12+13+14+16+16+18+18+20+23+23}{11} = 16.6 = 17$ [1]

Question 6a
March = 1.33

July = 0.76 (2 marks)

First find the seasonal average

$$seasonal\ average = \frac{28 + 26 + 24 + 17 + 16 + 15}{6} = 21$$

$$seasonal\ index = \frac{value\ for\ season}{seasonal\ average}$$

$$March = \frac{28}{21} = 1.33$$

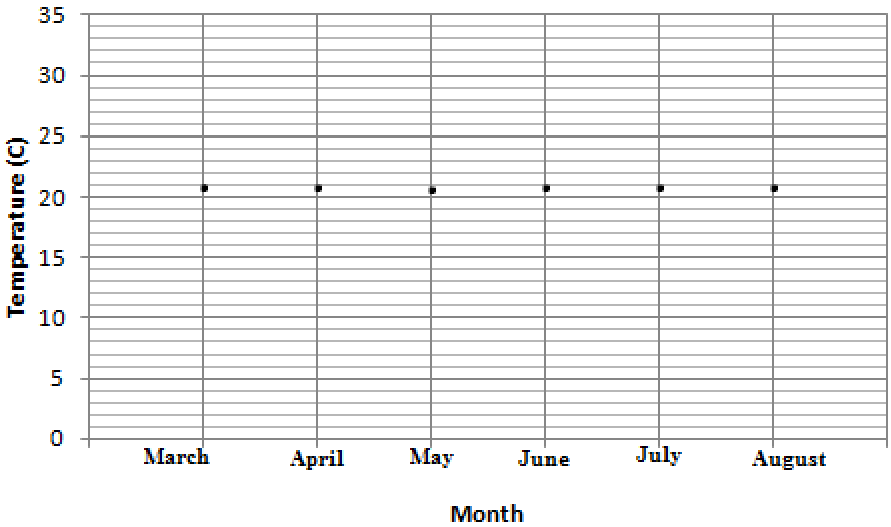
$$July = \frac{16}{21} = 0.76$$

Question 6b

$$deseasnalised\ value = \frac{actual\ value}{seasonal\ index}$$

Month	March	April	May	June	July	August
Deseasonalised temperature	$\frac{28}{1.33} = 21.05$	$\frac{26}{1.24} = 20.97$	$\frac{24}{1.14} = 21.05$	$\frac{17}{0.81} = 20.99$	$\frac{16}{0.76} = 21.05$	$\frac{15}{0.71} = 21.13$

Question 6c



Question 6d

Deseasonalising the data has removed the seasonal trend [1].

Recursion and Financial Modelling

Question 7a

\$1200 – found by the Y intercept, as the Y intercept represents the time 0 (at sale) [1].

Question 7b

[2] Find two points on the graph that are easily identifiable e.g. (0,1200) and (5, 780). These indicate that at 0 years, the printer was worth \$1200, and at 5 years, the printer was worth \$780.

$$amount\ of\ depreciation\ over\ 5\ years = 1200 - 780 = 420$$

$$amount\ of\ depreciation\ per\ year = \frac{420}{5} = \$84$$

Question 7c

$initial\ value - (depreciation\ \times n\ years) = \ book\ value\ after\ n\ years$

$1200 - (84 \times 7) = \$612$

Question 8a

25 cents [1], as seen in the equation

Question 8b

$Flat\ rate\ depreciation\ over\ 3\ years = 1200 - 3 \times 84 = \$948$

$948 = 1200 - 0.25 \times n$

$n = 1008$

As n is per 100 pages, $n = 1008\ \times 100 = 100{,}800\ pages$ [1]

Question 8c

Flat rate:

$V = 1200 - 84 \times 6 = \$696$

Unit cost:

$V = 1200 - 0.25 \times (300 \times 6) = \$750$

[1]

They should choose the unit cost method of depreciation [1]

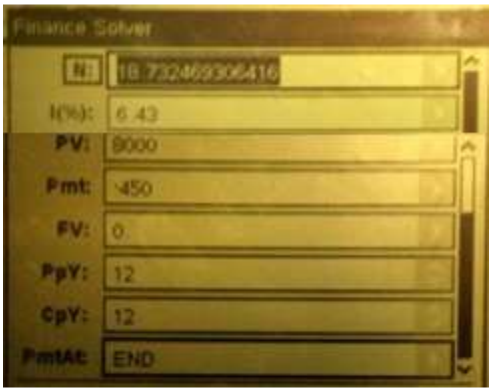
Question 9a

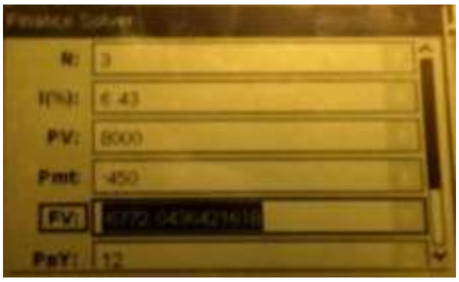
18.73 = 19 repayments [1]

Question 9b

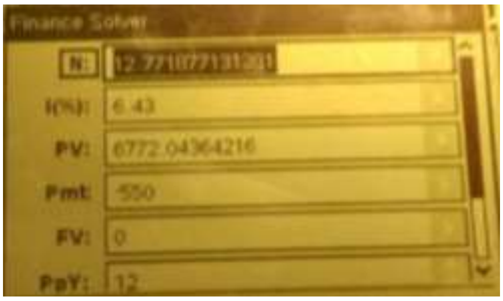
12.77 = 13 repayments

First, calculate the value of the loan after the 3 repayments of \$450.





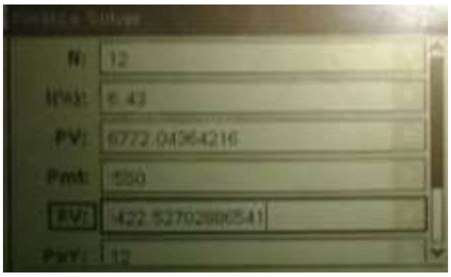
Then change the present value and payment value to find the number of further repayments.



Question 9c

$424.79 = $425 [1]

Calculate the amount left after 12 further payments.



Insert this new amount into present value, and calculate the final payment.

Section B – Modules

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Module 1- Matrices

Question 1a

$$E = \begin{array}{c} \begin{array}{cccc} L & M & S & P \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ L \\ M \\ S \\ P \end{array}$$

This is constructed as an arrow pointing to a team represents dominance over that team from the origin of the arrow. The matrix goes from rows to columns i.e. Magpies beat Leopards, therefore there is a 1 in row 2, column 1 to show the Magpies are dominant over Lions.

Question 1b i

The Stallions are dominant over the Magpies.

Question 1b ii

The Leopards are not dominant over any other team.

Question 1c

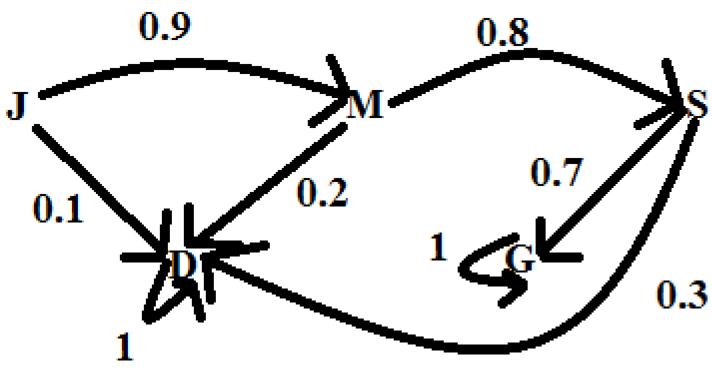
$$C = \begin{array}{c} \begin{array}{ccccc} L & M & P & S & T \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ L \\ M \\ P \\ S \\ T \end{array}$$

Question 2a

40. Drop-out rate is indicated in the transition matrix at row 4 column 1 = 0.1. Hence, 10% of 400 junior students.

Question 2b

Complete the transition diagram below, showing the relevant proportions [2].



Question 2c

$$S0 = \begin{bmatrix}400\\350\\250\\0\\0\end{bmatrix}\begin{matrix}J\\M\\S\\D\end{matrix}$$

The initial state matrix is determined from the information in the 3rd sentence of question 2. It is assumed there are no drop outs or graduates before the school starts.

Question 2d

$$TxS0 = S1 = \begin{bmatrix}0\\360\\280\\185\\175\end{bmatrix}\begin{matrix}J\\M\\S\\D\\G\end{matrix}$$

Question 2e

$$TxS1 = S2 = \begin{bmatrix}0\\0\\288\\341\\371\end{bmatrix}$$ = 288 students will remain at the school

Question 2f

3 years

$$T^3xS0 = S3 = \begin{bmatrix}0\\0\\0\\427.4\\572.6\end{bmatrix}$$ Therefore when n=3, there will be no more students at the school

Question 2g

$$S3 = \begin{bmatrix}400\\360\\288\\579\\573\end{bmatrix}\begin{matrix}J\\M\\S\\D\\G\end{matrix}$$

$$S1 = TS0 + 400J = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.9 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0.3 & 1 & 0 \\ 0 & 0 & 0.7 & 0 & 1 \end{bmatrix} \begin{bmatrix} 400 \\ 350 \\ 250 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 400 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 400 \\ 360 \\ 280 \\ 185 \\ 175 \end{bmatrix}$$

$$S2 = TS1 + 400J = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.9 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0.3 & 1 & 0 \\ 0 & 0 & 0.7 & 0 & 1 \end{bmatrix} \begin{bmatrix} 400 \\ 360 \\ 280 \\ 185 \\ 175 \end{bmatrix} + \begin{bmatrix} 400 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 400 \\ 360 \\ 288 \\ 381 \\ 371 \end{bmatrix}$$

$$S3 = TS2 + 400J = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.9 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0.3 & 1 & 0 \\ 0 & 0 & 0.7 & 0 & 1 \end{bmatrix} \begin{bmatrix} 400 \\ 360 \\ 288 \\ 381 \\ 371 \end{bmatrix} + \begin{bmatrix} 400 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 400 \\ 360 \\ 288 \\ 579 \\ 573 \end{bmatrix}$$

Module 2 – Networks and Decision Mathematics

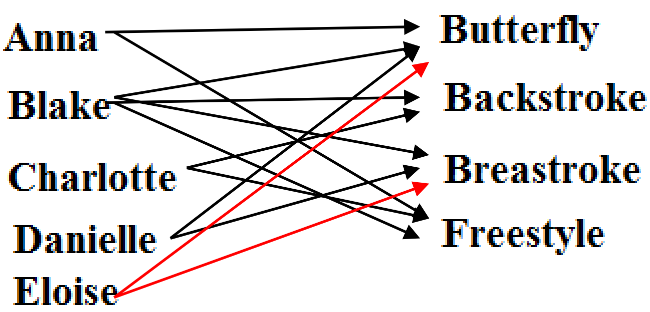
Question 1a

2 – Blake and Charlotte both have arrows pointing to backstroke, meaning they swim it

Question 1b

Breastroke – Blake and Danielle are the only people with arrows pointing to breastroke.

Question 1c



Question 2a

The first step in the Hungarian Algorithm is row reduction, where the smallest number (i.e. fastest time) in each row is taken away from all numbers in that row [1].

Task	Anna	Blake	Charlotte	Danielle
Butterfly	0	0	1	0
Backstroke	0	2	5	3
Breastroke	0	2	3	2
Freestyle	0	2	2	1

Question 2b

The minimum number of lines to cover all zeros after row and column reduction is less than four [1]. There needs to be at least the same number of lines as there are people for an allocation to be made using the Hungarian algorithm.

Question 2c
After completing a row and column reduction, finding the smallest uncovered element and subtracting that from every uncovered element, the following table should be reached.

Stroke	Anna	Blake	Charlotte	Danielle
Butterfly	0	0	0	0
Backstroke	0	1	3	2
Breastroke	0	1	1	1
Freestyle	3	1	0	0

Question 2d
Butterfly [1] as it is the only stroke with a '0' in her column.

Question 2e
33+36+45+30 = 144 seconds = 2 min 24 seconds [1].

Question 3a
13 hours [1].

For activity J to start, activity B (5 hours) and H (8 hours) must be complete. Therefore, it will be at least 13 hours before activity J can start.

Question 3b
14 hours [1].

As the minimum time for all activities to be completed is 21 hours and activity I takes 7 hours to complete, it can start as late as 14 hours into the project and all activities will still be completed by 21 hours.

$21 - 7 = 14$

Question 3c
4 hours [1].

$float\ time = latest\ start\ time - earliest\ start\ time$

$float\ time\ of\ activity\ I = 14 - 10 = 4\ hours$

The earliest start time is 10 hours, as activity B (5 hours) and activity F (5 hours) must be complete before activity I can start.

Question 3d
3 hours = $150 [1].

At the start, the critical path is BFGK =21 hours. There are 2 paths coming in 2nd at 18 hours – ACEK and ADGK. By crashing G by 3 hours, BFGK is reduced to 18 hours, the same as ACEK i.e. the new critical path and minimum completion time.

Module 3 – Geometry and Measurement

Question 1a

45° [1].

Question 1b

Triangle AXY is a right-angled isosceles (45 degrees):

Hence, AX = $\sqrt{2 \times 0.55\text{^}2} = 0.78$m [1].

Question 1c

Using cosine rule for triangle AXC,

XC$\sqrt{0.78^2 + 1.42^2 - 2 \times 0.78 \times 1.42 \times \cos(45)} = 1.03$m [1].

Question 1d

Using Heron's formula – calculate half of the triangles perimeter (s), then the area

$$s = \frac{2.51 + 1.42 + 1.81}{3} = 1.913$$

$A = \sqrt{1.913(1.913 - 2.51)(1.913 - 1.42)(1.913 - 1.81)} = 1.26m$ [1].

Question 1e

45° [1].

Question 1f

$$A = \frac{length\ of\ base + length\ of\ top}{2} \times height$$

$A = \frac{90+45}{2} \times 30 = 2025cm^2$ [1].

Question 2a

$\frac{2.4cm-(2\times0.1cm)}{2} = 1.1cm$ [1].

Question 2b

$V = \pi r^2 h$

$V = \pi \times 1.1^2 \times 1.5 = 5.7m^3$ [1].

Question 2c

$external\ surface\ area = area\ of\ base + area\ of\ top + area\ of\ sides$

$external\ surface\ area = 1.2^2 \times \pi + \ 1.2^2 \times \pi + \ 2.4 \times \pi \times 1.7 = 21.87$ [1].

Question 2d

$external\ surface\ area\ without\ base = 1.2^2 \times \pi + \ 2.4 \times \pi \times 1.7 = 17.34 = 17.3$ [1].

Question 2e

$17.3 \times 27 = \$467.10$ [1].

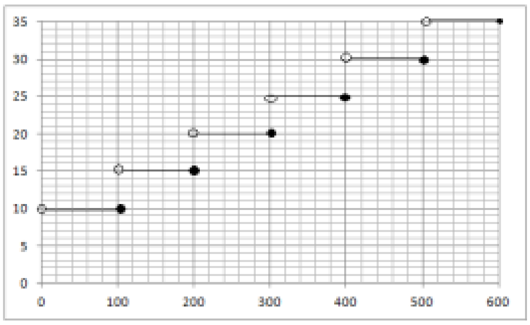
Module 4 -Graphs and Relations

Question 1a

101km [1].

This is determined from Table 1.

Question 1b



Question 1c

$fare = 3.95 + 0.05 \times 270 = 17.45$

$fare\ difference = graph\ fare - equation\ fare = 20 - 17.45 = \$2.55$

Question 1d

600 km on the graph = \$35

600 km in the equation = $3.95 + 0.05 \times 600 = \$33.95$

Therefore, it is cheaper to use the equation as a fare plan.

Question 1e

Solve 2 simultaneous equations selected from

10 = a+ 100b

15 = a +200b

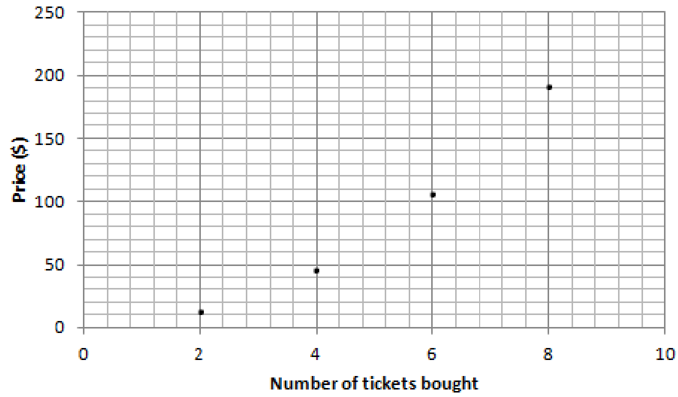
20 = a+ 300b

25 = a + 400b

30 = a + 500b

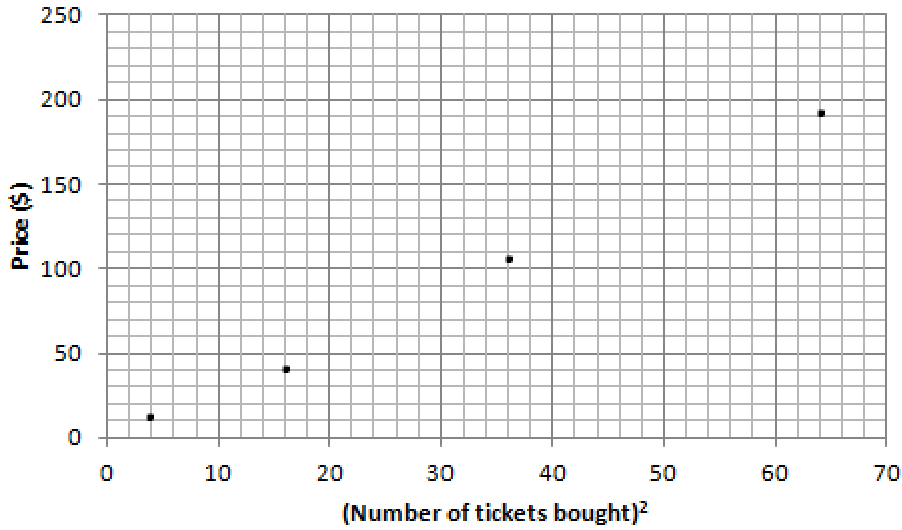
A = 5, B = 1/20 = 0.05

Question 2a



Question 2b

Number of tickets bought	2	4	6	8
(Number of tickets bought)^2	4	16	$6^2 = 36$	$8^2 = 64$
Price ($)	12	48	108	192



Question 2c

K=3

Question 2d

From question 1a, a trip between 500-600km costs $35, therefore 12x$35 = $420

From question 2c, 12 trips of any distance costs 3×12^2=$432 [1].

Therefore, it is cheaper to buy the tickets separately [1], rather than buying the multi-trip ticket.