

Engage  
Education  
Foundation

# **Specialist Mathematics**

Notes Solutions

**Question 1**

(a)  $\frac{-\pi}{6}$

(b)  $\frac{243}{4}$

(c)  $\frac{-3\pi}{4}$

(d)  $c = 3$

**Question 2**

(a)  $2\sqrt{5}cis\left(\frac{-2\pi}{3}\right)$

(b)  $\sqrt{14}cis\left(\frac{-3\pi}{4}\right)$

(c)  $\frac{49}{10\sqrt{5}}cis(\pi)$

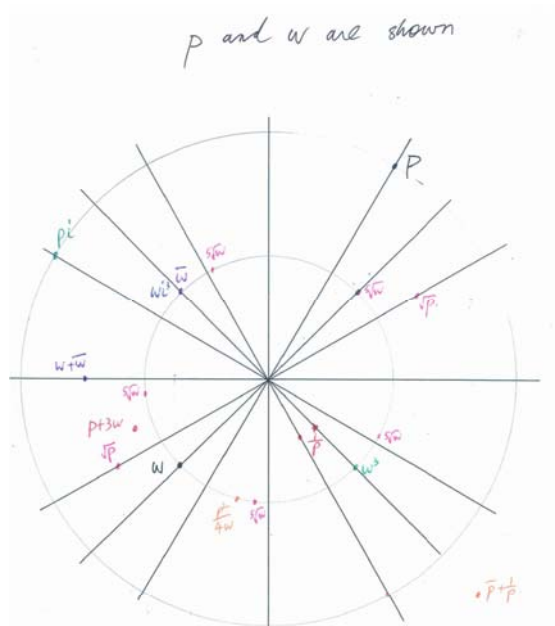
(d)  $\frac{-49}{10\sqrt{5}}i$

**Question 3**

$\pm(2-5i)$

**Question 4**

P and w are shown.

**Question 5**

$$z = 3 + \sqrt{5}i, z = 1$$

**Question 6a**

$$\sqrt{i} = \pm \frac{1}{\sqrt{2}}(1+i)$$

**Question 6b**

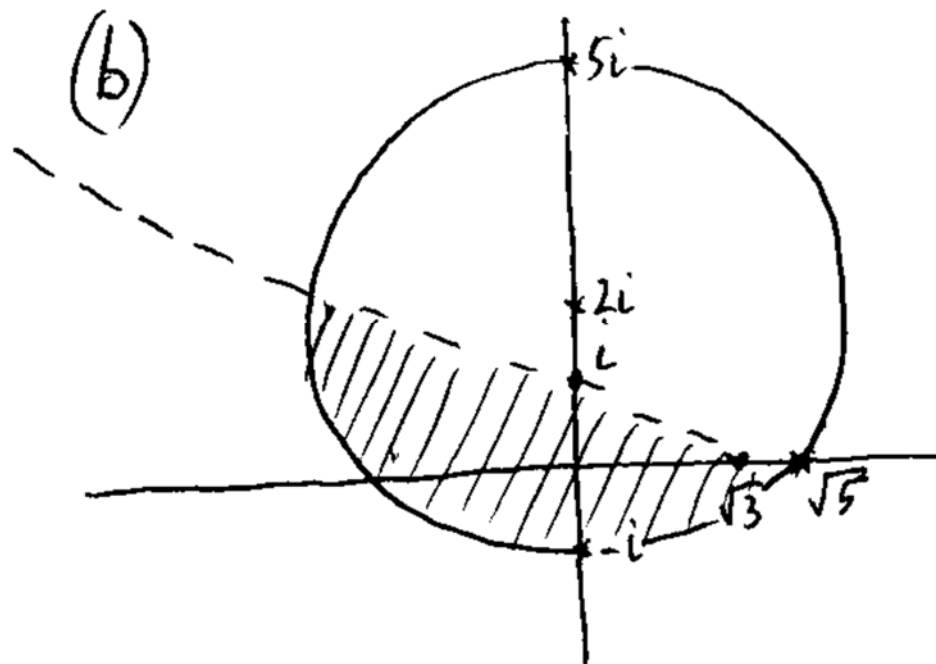
$$\begin{aligned}\Delta &= (2-4i)^2 - 4 \times 1 \times (-3-6i) \\ &= 4 - 16i - 16 + 12 + 24i \\ &= 8i\end{aligned}$$

**Question 6c**

$$\begin{aligned}z &= \frac{-(2-4i) \pm \sqrt{8i}}{2} \\ &= -1 + 2i \pm \frac{2\sqrt{2i}}{2} \\ &= -1 + 2i \pm \sqrt{2} \times \frac{1}{\sqrt{2}}(1+i) \\ &= -1 + 2i \pm (1+i) \\ &= 3i, -2+i\end{aligned}$$

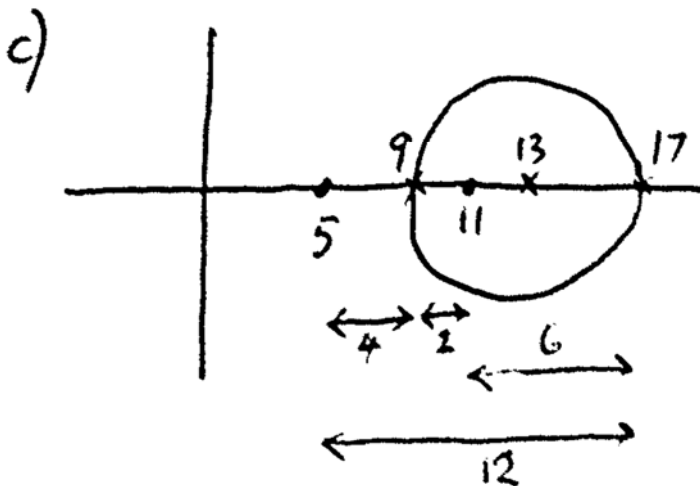
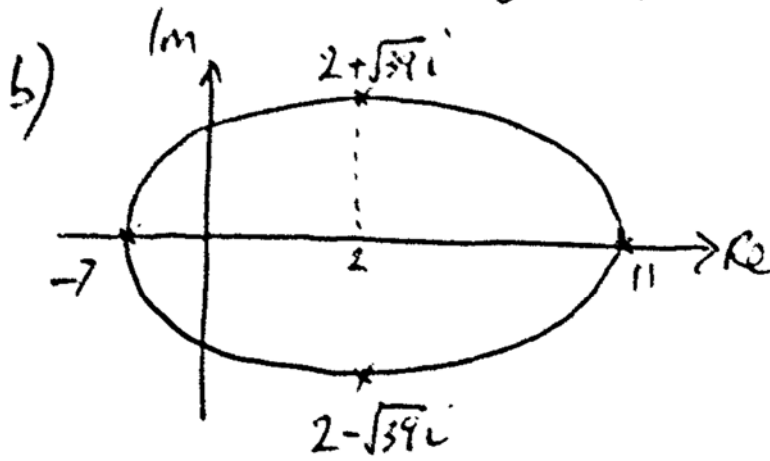
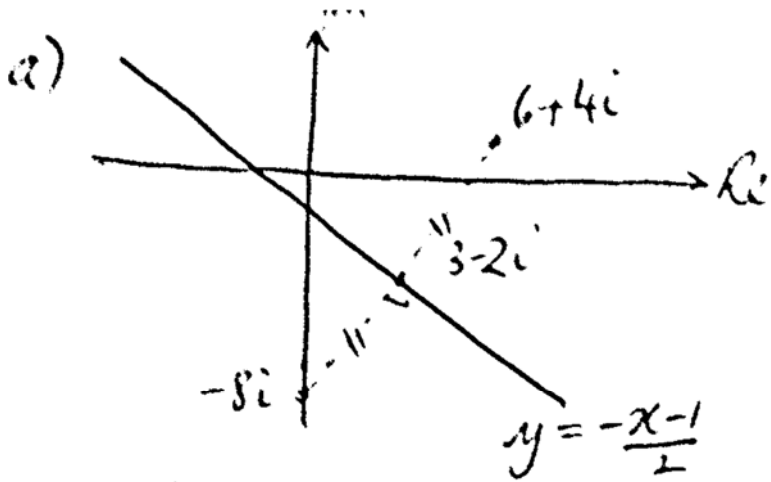
**Question 7a**

Circle, centre (3,-4), radius 5

**Question 7b****Question 7c**

$$z = \frac{-\sqrt{3}}{4}(1 + \sqrt{17}) + \frac{i}{4}(5 + \sqrt{17})$$

Question 8



## Question 9

$w = 1cis\theta$  where  $\theta = \frac{n\pi}{5}$ ,  $n$  is an integer which is NOT a multiple of 5

## Question 10

(a)  $cz^2 + bz + a$

(b)  $c^2z^2 + (2ac - b^2)z + a^2$

## Question 11

Proof

## Question 12

$$\overline{AK} = \frac{1}{7}(4\mathbf{t} + 2\mathbf{u})$$

## Question 13a

$38^\circ$

## Question 13b

$$\frac{11}{14} \left( \begin{matrix} i \\ \% \end{matrix} - 2 \begin{matrix} j \\ \% \end{matrix} + 3 \begin{matrix} k \\ \% \end{matrix} \right)$$

## Question 14

(a) show

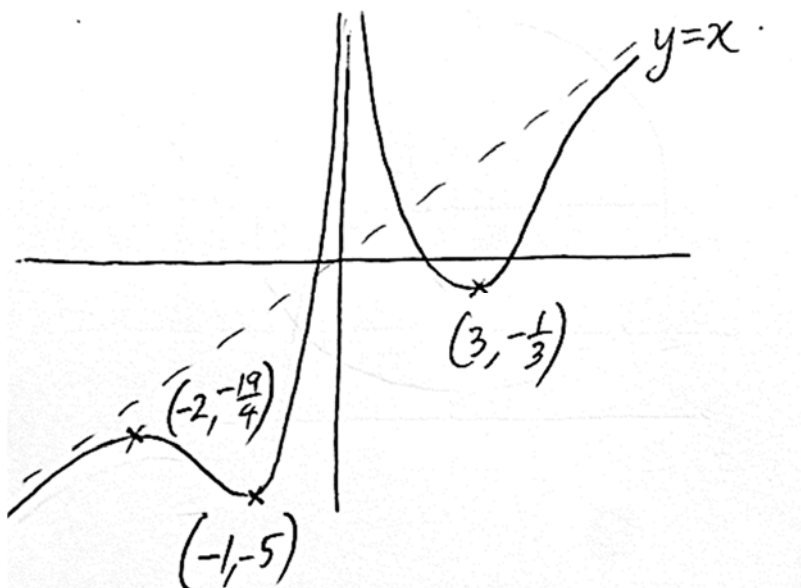
(b)  $y = \pm(1 - 4x^2)\sqrt{1 - x^2}$

(c) it is the ABC logo!

## Question 15

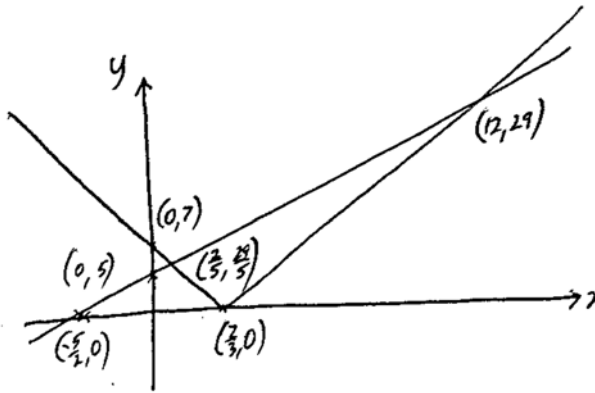
The portion of  $y = (x - 6)^3 - 1$  from  $(6, -1)$  to  $(7, 0)$

## Question 16



## Question 17

Intersect at  $(12, 29)$  and  $(\frac{2}{5}, \frac{29}{5})$



## Question 18

either :  $(x+2)^2 - 9 = x+5$

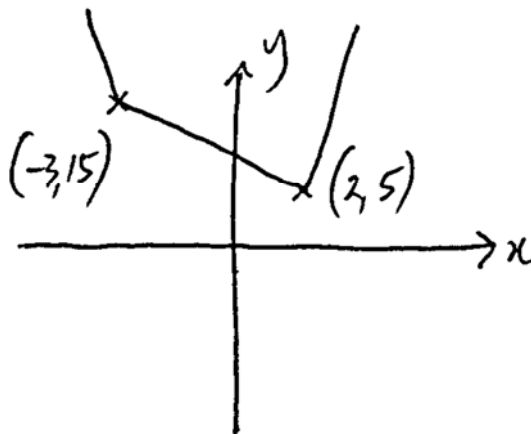
$x = -5, 2$

or :  $-(x+2)^2 + 9 = x+5$

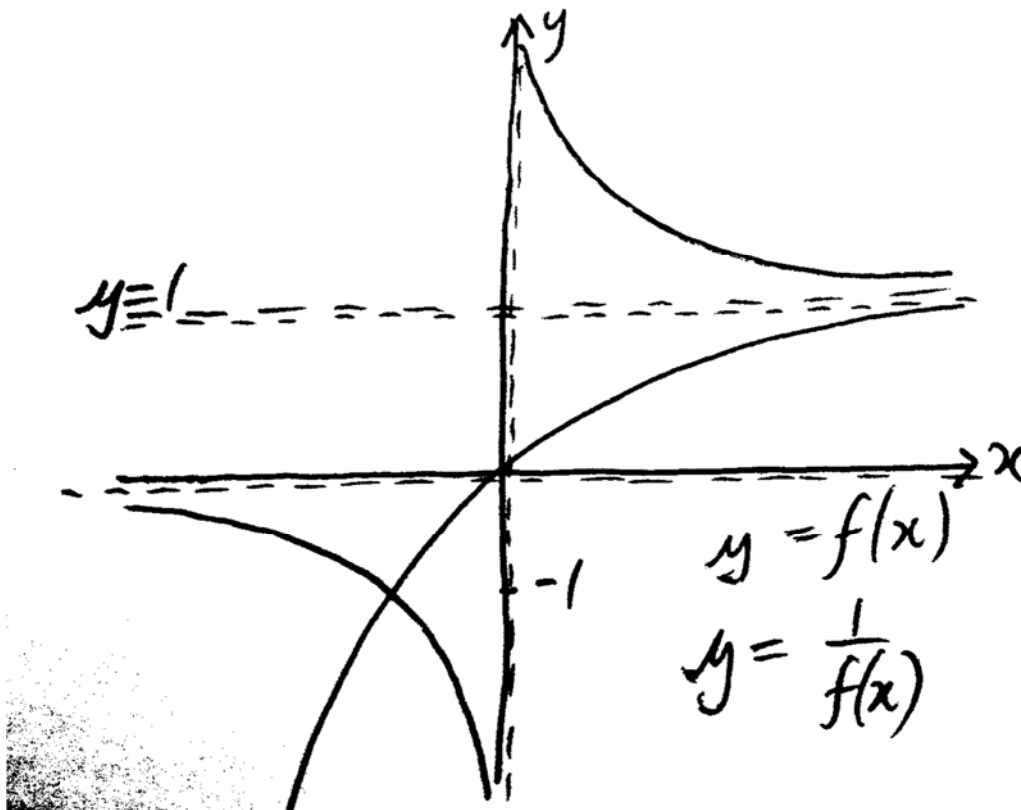
$x = -5, 0$

overall :  $x = -5, 0, 2$

## Question 19



## Question 20



## Question 21

- (a)  $2 \cos \theta - 1$   
 (b)  $-2 \cot x$

## Question 22

- (a)  $\frac{5\pi}{6}$   
 (b)  $-1$   
 (c)  $\frac{7\pi}{11}$   
 (d)  $\frac{\sqrt{9-a^2}}{3}$   
 (e)  $\frac{1}{\sqrt{2}}$   
 (f)  $1$   
 (g)  $\frac{2\sqrt{6}}{7}$   
 (h)  $\frac{-1}{3}$



## Question 23

$$(a) y = 50 \sin\left(\frac{\pi x}{100}\right)$$

$$y = \frac{-100}{\pi} \sin^{-1}\left(\frac{x}{50}\right)$$

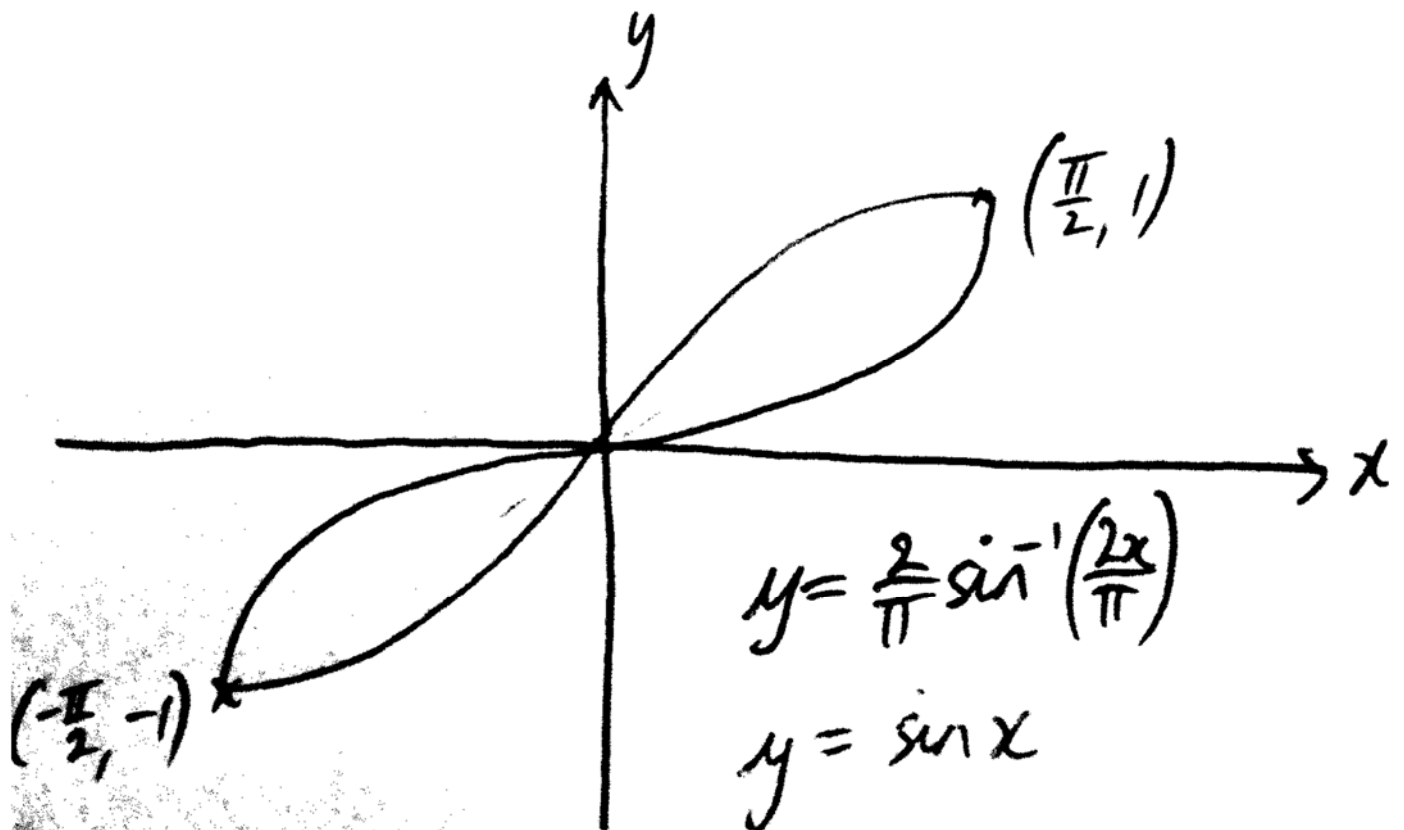
$$(b) \frac{69\sqrt{2}\pi}{5} \text{ km/h}$$

## Question 24

(a) show

$$(b) x = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$$

## Question 25

Curves both pass through  $\left(-\frac{\pi}{2}, -1\right), (0,0), \left(\frac{\pi}{2}, 1\right)$ 

## Question 26

$$(a) t^5 - \frac{45t^4}{4} + c$$

$$(b) \int 5t + 45 + \frac{405}{t-9} dt = \frac{5t^2}{2} + 45t + 405 \ln|t-9| + c$$

$$(c) \text{let } u = t-9: \int 5(u+9) \cdot u^{-1/2} du = \frac{10u^{3/2}}{3} + 90u^{1/2} + c = \frac{10(t-9)^{3/2}}{3} + 90(t-9)^{1/2} + c$$

$$(d) \frac{5}{2} \ln(t^2 + 9) + c$$

$$(e) \frac{5}{3} \tan^{-1}\left(\frac{t}{3}\right) + c$$

$$(f) \frac{5}{6} \int \frac{1}{t-3} - \frac{1}{t+3} dt = \frac{5}{6} \ln \left| \frac{t-3}{t+3} \right| + c$$

$$(g) 5\sqrt{t^2 - 9} + c$$

$$(h) \frac{5}{2} \ln|t^2 - 9| + c$$

$$(i) 5 \sin^{-1}\left(\frac{t}{3}\right) + c$$

$$(j) 5 \int \frac{1 - \cos(18x)}{2} dx = \frac{5x}{2} - \frac{5 \sin(18x)}{36} + c$$

$$(k) \int \frac{5 \sin(9x)}{\cos(9x)} dx = \frac{5}{-9} \ln|\cos(9x)| + c$$

$$(l) \int 9 \sec^2(5x) dx = \frac{9}{5} \tan(5x) + c$$

$$(m) \frac{\sin^6(9x)}{54} + c$$

$$(n) \int \sin(5x) \cdot \sin^2(5x) \cdot \cos^2(5x) dx = \int \sin(5x) \cdot \cos^2(5x) - \sin(5x) \cdot \cos^4(5x) dx = \frac{\cos^3(5x)}{-15} + \frac{\cos^5(5x)}{25} + c$$

$$(o) \frac{\tan^6(9x)}{54} + c$$

$$(p) \frac{1}{5} \int (\ln(9x))^1 \cdot \frac{1}{x} dx = \frac{(\ln(9x))^2}{10} + c$$

## Question 27

$$x = \cos u, \quad \sqrt{1-x^2} = \sin u, \quad dx = -\sin u \cdot du$$

$$\int \frac{\cos^3 u}{\sin u} \cdot -\sin u \cdot du = \int -\cos^3 u \cdot du = \int (\sin^2 u - 1) \cos u \cdot du = \frac{\sin^3 u}{3} - \sin u + c$$

$$= \frac{(1-x^2)^{3/2}}{3} - \sqrt{1-x^2} + c$$

**Question 28**

$$y'' = a(x - 2)$$

$$y' = a\left(\frac{x^2}{2} - 2x + b\right)$$

$$y'(-1) = 0 \Rightarrow b = \frac{-5}{2}$$

$$y' = a\left(\frac{x^2}{2} - 2x - \frac{5}{2}\right)$$

$$y = a\left(\frac{x^3}{6} - x^2 - \frac{5x}{2} + c\right)$$

$$y(2) = 49, y(-1) = 5$$

$$\Rightarrow a = 6, c = \frac{-1}{2}$$

$$y = x^3 - 6x^2 - 15x - 3$$

**Question 29a**

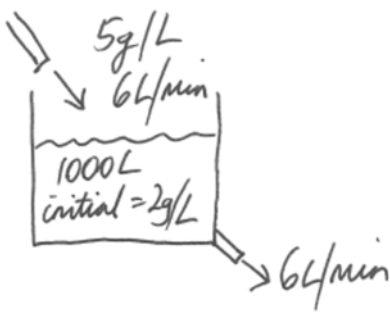
$$\int \frac{dy}{e^y} = \int e^{2x} dx$$

$$-e^{-y} = \frac{e^{2x}}{2} + c, c = \frac{-3}{2}$$

$$y = -\log_e\left(\frac{3 - e^{2x}}{2}\right)$$

**Question 29b**

(b) Domain =  $(-\infty, \log_e \sqrt{3})$

**Question 30**

$$\frac{dQ}{dt} = \text{rate in} - \text{rate out}$$

$$= 30 - \frac{6Q}{1000}$$

$$\Rightarrow \frac{dt}{dQ} = \frac{1000}{30000 - 6Q}$$

$$t = \frac{1000}{-6} \int \frac{-6}{30000 - 6Q} dQ$$

$$= \frac{1000}{-6} \ln|30000 - 6Q| + c$$

$$\begin{cases} t = 0 \\ Q = 2000 \end{cases} \Rightarrow t = \frac{1000}{-6} \ln \left| \frac{30000 - 6Q}{18000} \right|$$

$$(a) \quad Q = 5000 - 3000e^{-\frac{6t}{1000}}$$

$$t = 30 \Rightarrow Q = 2494, \text{concentration} = 2494 \text{g/L}$$

$$(b) \quad Q = 4000 \Rightarrow t = 183.1 \text{ min}$$

(c) Sugar poured out = initial sugar + sugar in - sugar left

$$2000 + 183.1 \times 30 - 4000$$

$$= 3493 \text{g}$$

### Question 31

$$(a) \quad \frac{dv}{dt} = \frac{k}{t+1}$$

$$V = kh|t+1| + c$$

$$\begin{cases} t = 0 \\ v = 0 \end{cases} \Rightarrow c = 0$$

$$\begin{cases} t = 10 \\ v = 2000 \end{cases} \Rightarrow k = \frac{2000}{\ln 11}$$

$$\Rightarrow V = \frac{2000}{\ln 11} \ln |t + 1|$$

$$(b) \quad V = 1000 \rightarrow t = 2.3166$$

$$(c) \quad t = 5 \rightarrow V = 1494.4 \text{ml}$$

### Question 32

$$(a) \quad \frac{dV}{dt} = -kV^2 \sqrt{t}$$

$$\int \frac{dV}{V^2} = \int -k\sqrt{t} dt$$

$$\frac{-1}{V} = \frac{-2kt^{3/2}}{3} + c, c = \frac{-1}{56}$$

$$\text{given: } t = 9, V = 28$$

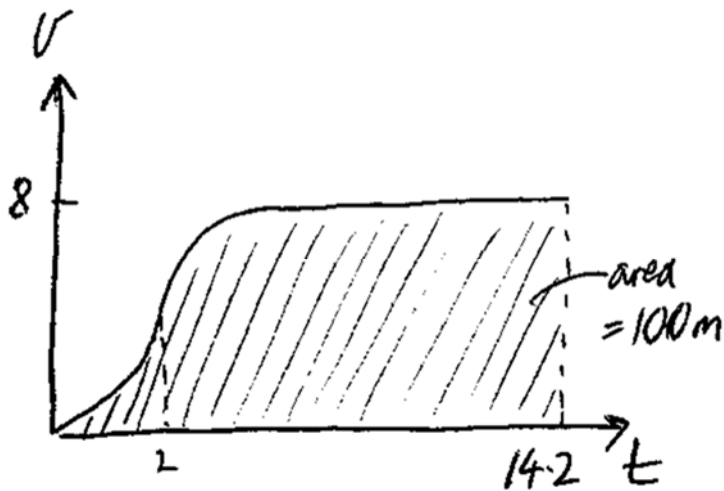
$$(b) \Rightarrow k = \frac{1}{56 \times 18}$$

$$\frac{1}{V} = \frac{1}{56} \left(1 + \frac{t^{3/2}}{27}\right)$$

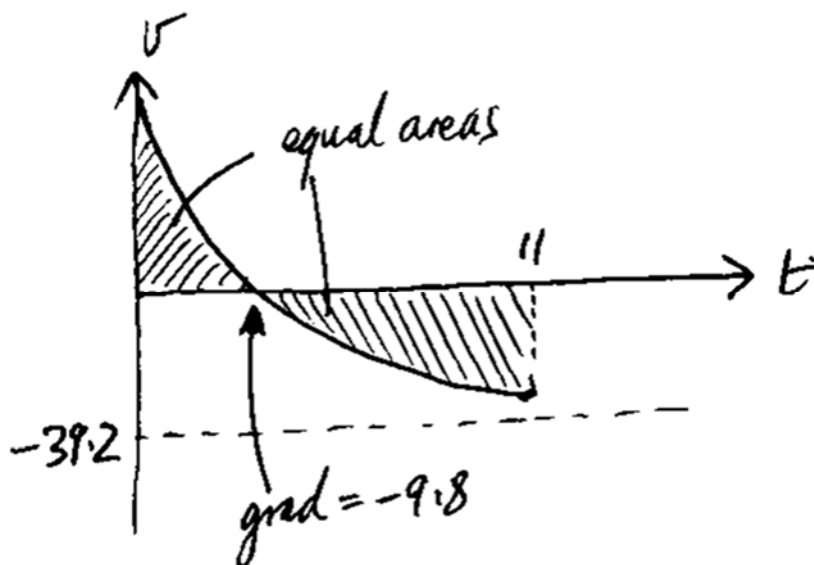
$$t = 9, \frac{1}{V} = \frac{1}{56} (1 + 27) = \frac{1}{2}$$

$$\Rightarrow V = 2$$

Question 33a



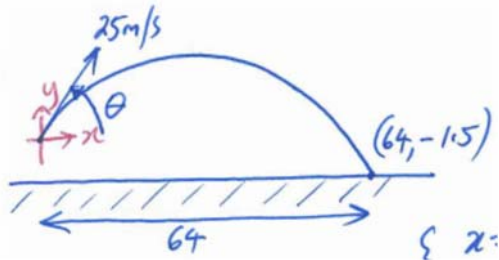
Question 33b



## Question 34

- (a) H
- (b) I
- (c) C
- (d) F
- (e) A

## Question 35a

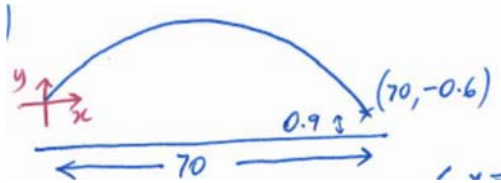


$$y = x \tan \theta - \frac{gx^2}{2u^2}(\tan^2 \theta + 1)$$

$$\begin{cases} x = 64 \\ y = -1.5 \end{cases} \Rightarrow \tan \theta = 0.7972, 1.1956$$

$$\Rightarrow \theta = 38.56^\circ \text{ or } 50.09^\circ$$

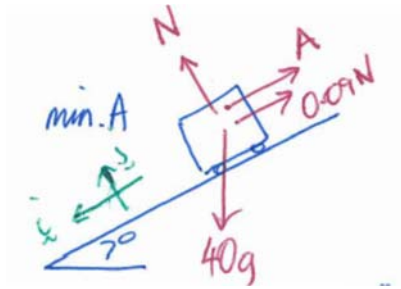
## Question 35b



$$y = x \tan \theta - \frac{gx^2}{2u^2}(\tan^2 \theta + 1)$$

$$\begin{cases} x = 70 \\ y = -0.6 \end{cases} \Rightarrow \text{no solution, impossible}$$

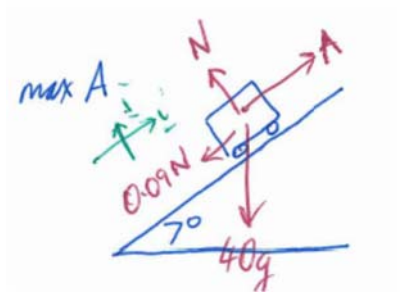
## Question 36



$$\sum F_j = 0 = N - 40g \cos 7^\circ$$

$$\sum F_i = 0 = 40g \sin 7^\circ - A - 0.09N$$

$$\Rightarrow A = 12.76N$$



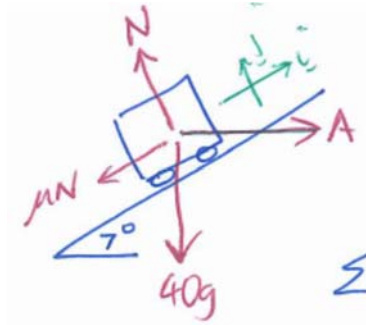
$$\sum F_j = 0 = N - 40g \cos 7^\circ$$

$$\sum F_i = 0 = A - 40g \sin 7^\circ - 0.09N$$

$$\Rightarrow A = 82.79N$$

$$\Rightarrow \text{Any force} \in [12.76, 82.79]$$

## Question 37



$$\begin{aligned} u &= 0 \\ v &= 1.5 \\ x &= 10 \end{aligned}$$

$$\text{use } 2ax = v^2 - u^2$$

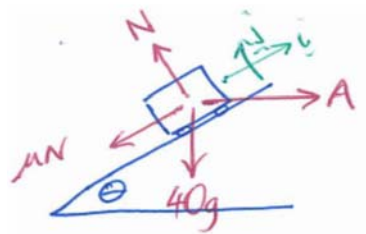
$$\Rightarrow a = \frac{1.5^2}{20}$$

$$\sum F_j = 0 = N - 40g \cos 7^\circ - A \sin 7^\circ$$

$$\sum F_i = 40 \times \frac{1.5^2}{20} = A \cos 7^\circ - A \sin 7^\circ$$

$$\text{Solve for } A: A = 88.93\text{N}$$

## Question 38



$$A = 400 - 20x$$

$$x = 20, v = 2\text{m/s}$$

$$\sum F_j = 0 = N - 40g \cos \theta - (400 - 2x) \sin \theta$$

$$\sum F_i = 40a(x) = (400 - 2x) \cos \theta - 40g \sin \theta - 0.09N$$

$$\Rightarrow a(x) = (364.72 \cos \theta - 428 \sin \theta) + x(-20 \cos \theta + 0.18 \sin \theta)$$

$$a = \frac{d(\frac{1}{2}v^2)}{dv}$$

$$\Rightarrow \frac{1}{2}v^2 = (364.72 \cos \theta - 428 \sin \theta)x + \frac{x^2}{2}(-20 \cos \theta + 0.18 \sin \theta) + c$$

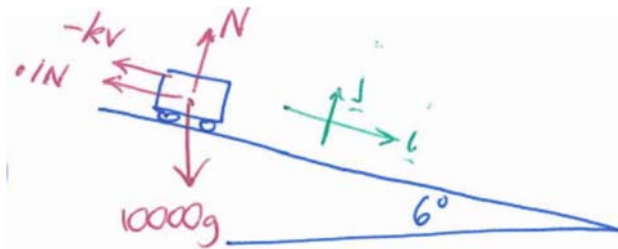


$$\text{When } \begin{cases} x = 0 \\ v = 2 \end{cases} \Rightarrow c = 0$$

$$\text{sub } \begin{cases} x = 20 \\ v = 2 \end{cases} \Rightarrow \theta = 21.124^\circ$$

### Question 39

(a)



$$(b) \sum F_j = 0 = N - 10000g \cos 6^\circ$$

$$\sum F_i = 10000a = 10000g \sin 6^\circ - 0.1N - kv$$

$$\Rightarrow a = g \sin 6^\circ - 0.1g \cos 6^\circ - \frac{kv}{10000} \approx \frac{497.4748 - kv}{10000}$$

$$(c) \frac{dt}{dv} = \frac{10000}{497.4748 - kv}$$

$$t = -\frac{10000}{k} \ln|497.4748 - kv| + c$$

$$\begin{cases} t = 0 \\ v = 0 \end{cases}$$

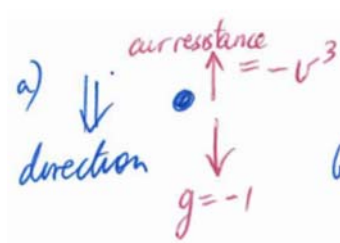
$$= -\frac{10000}{k} \ln \left| \frac{497.4748 - kv}{497.4748} \right|$$

$$\begin{cases} t = 25 \\ v = \frac{4}{3.6} \end{cases} \Rightarrow k = 0.01509$$

$$(d) a = 0 \Rightarrow v = \frac{497.4748}{k} = 32967 \text{ m/s}$$

### Question 40

(a)



$$(b) a = -1 - v^3$$

$$\Rightarrow v^3 = -1$$

$$\Rightarrow v = -1$$

$$(c) \quad r \frac{dv}{dx} = -1 - v^3$$

$$\Rightarrow \frac{dv}{dx} = \frac{1 - v^3}{-v} \Rightarrow \frac{dx}{dv} = \frac{-v}{1 - v^3}$$

$$(d) \quad x = \int \frac{-v}{1+v^3} dv = -\frac{1}{6} (\ln |v^2 - v + 1| - 2 \ln |v + 1| + 2\sqrt{3} \tan^{-1}(\frac{2v-1}{\sqrt{3}})) + c$$

$$\begin{cases} x = 0 \\ v = 0 \end{cases} \Rightarrow c = \frac{-\pi\sqrt{3}}{18}$$

$$(e) \quad v = -0.99 \Rightarrow x = -1.4159m$$