



# **Units 3 and 4 Mathematical Methods**

## **Notes Solutions**

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email [practiceexams@ee.org.au](mailto:practiceexams@ee.org.au).

**Question 1**

The correct answer is A.

**Question 2**

The correct answer is A.

**Question 3**

The correct answer is D.

**Question 4**

The correct answer is C.

**Question 5**

The correct answer is C.

**Question 6**

The correct answer is C.

**Question 7**

The correct answer is C.

**Question 8**

$$y = \frac{-2}{3}x + \frac{14}{3}$$

**Question 9**

The correct answer is E.

**Question 10**

The correct answer is C.

**Question 11**

The correct answer is E.

**Question 12**

$$\frac{1}{4a}(x-a)^2(x-b)(x+2a)^2$$

**Question 13a**

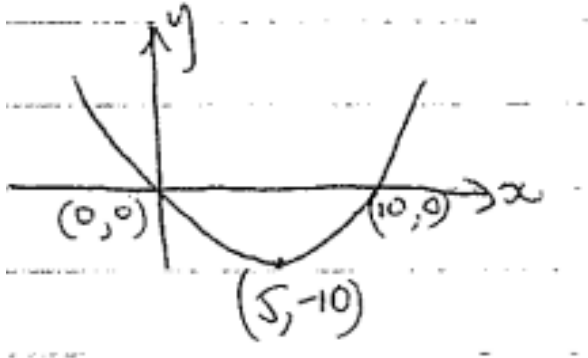
$$k = -4$$

**Question 13b**

$$k \in R \setminus \{-3, -4\}$$

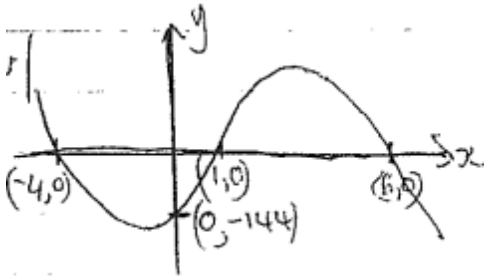
## Question 14a

$$y = \frac{2}{5}(x - 5)^2 - 10$$



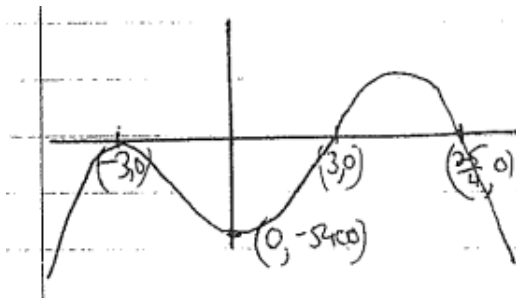
## Question 14b

$$y = -6(x - 1)(x - 6)(x + 4)$$

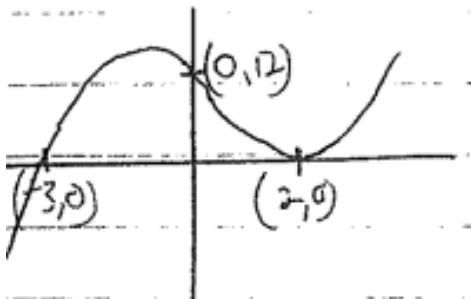


## Question 14c

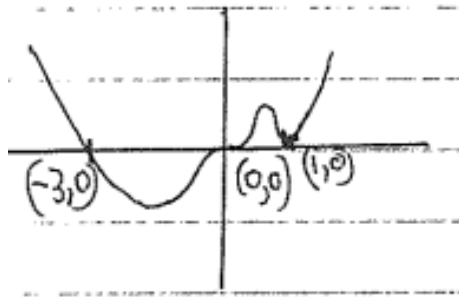
$$y = -32(x + 3)^2(x - 3)(x - \frac{25}{4})$$



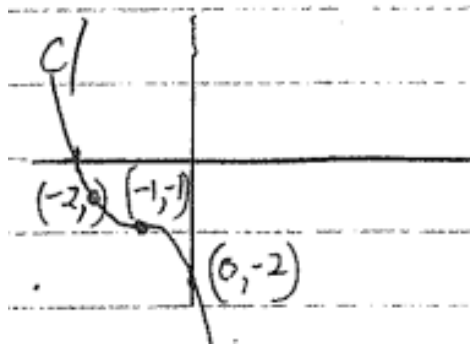
## Question 15a



**Question 15b**



**Question 15c**



**Question 16a**

$x \in R$

**Question 16b**

$x \in R \setminus \{-2\}$

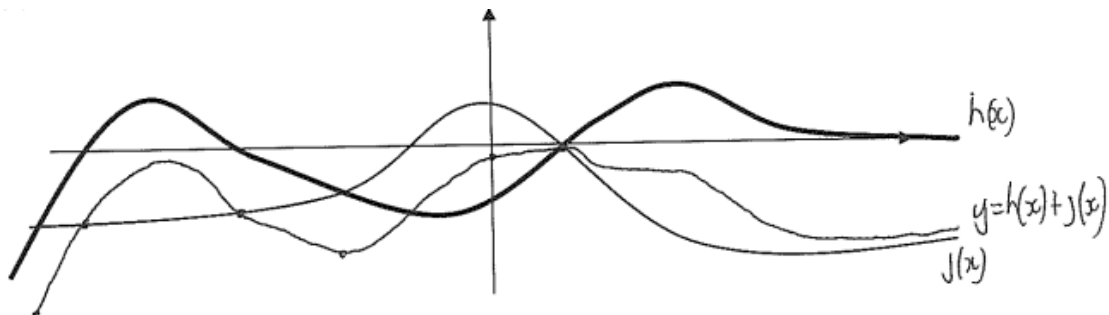
**Question 16c**

$x \in \left(-\infty, \frac{11}{2}\right] \setminus \left\{-4, \frac{7}{2}\right\} \notin$

**Question 17**

The correct answer is E.

**Question 18a**



**Question 19a**

$f(g(x)) = (\log_e(x))^2 - 4 \quad x \in (0, \infty)$

**Question 19b**

$g(f(x))$  does not exist since  $\text{range } f(x) \not\subseteq \text{dom } g(x)$

$$[-4, \infty) \not\subseteq (0, \infty)$$

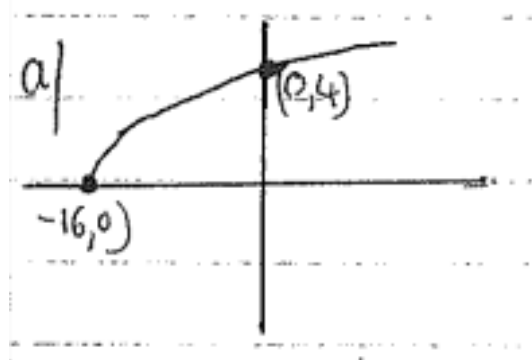
**Question 20a**

$$y_1 = \sqrt{\frac{-(x+2)}{2}} + 3 \quad x \in (-\infty, 2] \quad y_1 = [3, \infty)$$

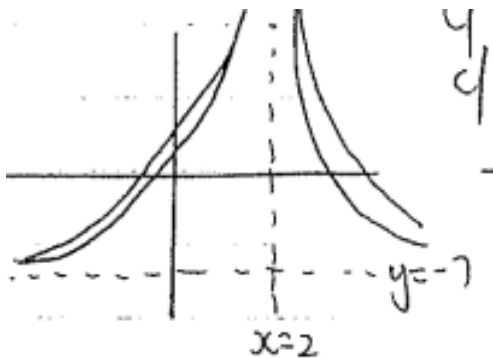
**Question 20b**

$$y_2 = \sqrt{\frac{-(x+2)}{2}} + 3$$

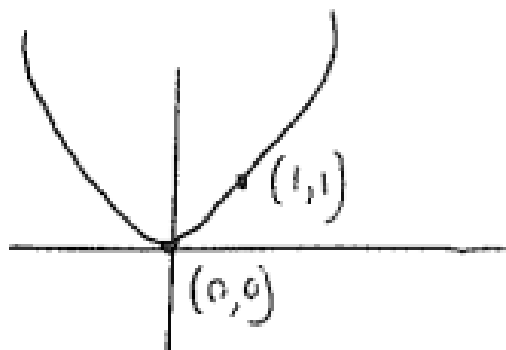
**Question 21a**



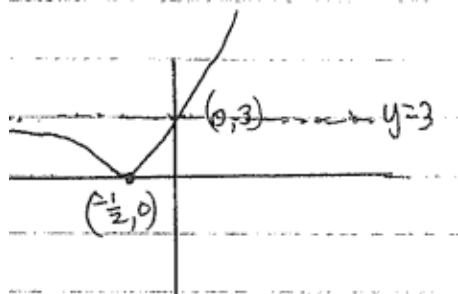
**Question 21b**



**Question 21c**



## Question 22



## Question 23

The correct answer is B.

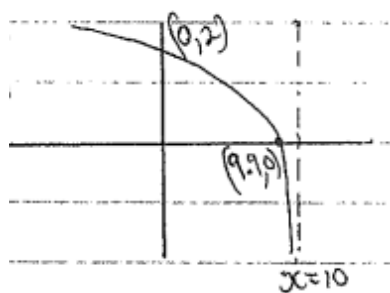
## Question 24

The correct answer is D.

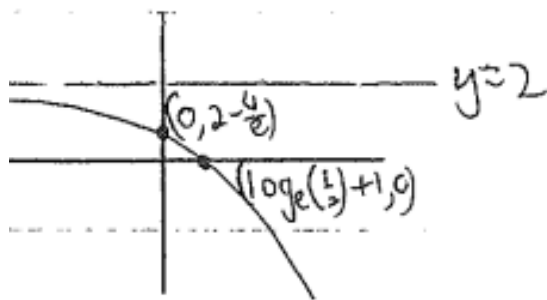
## Question 25

$\frac{3}{7}, 8, 16, 3, \frac{32}{9}$

## Question 26a



## Question 26b



## Question 27a

60g

## Question 27b

$\frac{60}{e}g$

## Question 27c

$4 \log_e(2)$

**Question 28**

The correct answer is E.

**Question 29a**

$$a = 2$$

**Question 29b**

$$w^{-1} = \sqrt{8-x} + 2, x \in (-\infty, 16]$$

**Question 30**

The correct answer is C.

**Question 31**

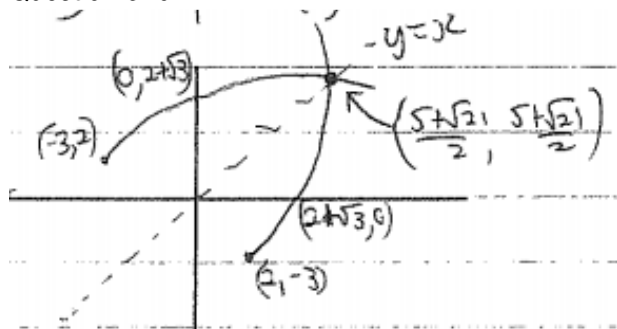
The correct answer is B.

**Question 32a**

$$(2, \infty)$$

**Question 32b**

$$m^{-1}(x) = \sqrt{x+3} + 2 \quad x \in [-3, \infty)$$

**Question 32c****Question 33**

The correct answer is E.

**Question 34**

$$12, 2, 16, 12$$

**Question 35**

$$x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{27\pi}{12}$$

**Question 36**

$$a = 4.5 \quad n = \frac{4}{3}$$

**Question 37**

$$\text{Max} = 29\text{m}$$

$$\text{Min} = 1\text{m}$$

$$\text{Period} = 90 \text{ sec}$$

**Question 38**

$$\frac{1}{3}$$

**Question 39**

$$\frac{dy}{dx} = -12x^2 \sin x^3 \quad \frac{d}{dx} = \frac{-1}{2\sqrt{1-x}}$$

$$f'(x) = \frac{-(\sin x + 2 \cos x)}{e^{2x}} \quad g'(x) = \frac{1 - \log_e(2x)}{2x^2}$$

$$\frac{dy}{dx} = 24x^{4x} \cos(3x) - 18e^{4x} \sin(3x) \quad \frac{dd}{dx} = 4x^3 \cos(x) - x^4 \sin(x)$$

**Question 40**

The correct answer is E.

**Question 41**

The correct answer is D.

**Question 42a**

$$\frac{1}{2}L$$

**Question 42b**

$$\frac{\pi}{4}L/h$$

**Question 43**

Question doesn't work

**Question 44**

$$x = \frac{11}{8}$$

$x$	1	$\frac{11}{8}$	3
$g'(x)$	-ve	0	+ve

$\therefore \setminus \_ /$  local minimum

**Question 45**

$$y = \frac{5x}{3} - \frac{16}{3}$$

**Question 46**

$$d - \frac{12}{\sqrt[3]{\pi}}$$

$$S = 108\sqrt[3]{\pi}$$

**Question 47**

The correct answer is C.



**Question 48**

$$\frac{x^3}{3} - \frac{3x^2}{2} + 2x + c \quad \frac{1}{10}(2x - 1)^5 + c$$

$$\frac{-\cos(5x + 2)}{5} + c$$

$$-2\sqrt{-3x + 4} + c$$

$$\frac{2}{\pi} \sin\left(\frac{\pi x}{2}\right) - 6 \log_e |6x + 5| + c$$

$$\frac{e^{2x}}{2} - \frac{e^{-2x}}{2} - 2x + c$$

**Question 49a**

981

**Question 49b**

6

**Question 50a**

0 and 7

**Question 50b**

$$\int_0^7 -x^2 + 5x + 14 dx$$

**Question 50c**

$$\frac{637}{6} \text{ sq units}$$

**Question 51**

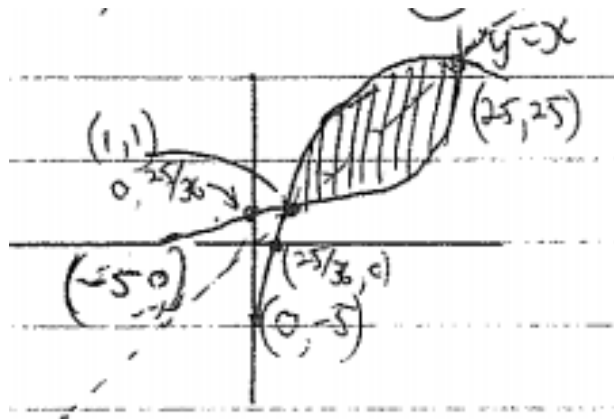
-4, 12, 8, 14, 4

**Question 52**

Question does not work

**Question 53a**

(1,1) and (25,25)

**Question 53b**

**Question 53c**

128 sq units

**Question 54a**

$$f'(x) = 2x \cos x - \cos x + 2 \sin x$$

**Question 54b**

$$x \sin x + \cos x + c$$

**Question 54c**

$$-\frac{\pi}{2} - 1$$

**Question 55a**

1000 sq units

**Question 55b**

1000 sq units

**Question 55c**

4000 sq units

**Question 56a**

$$\frac{3}{20}$$

**Question 56b**

$$\frac{3}{10}$$

**Question 57**

The correct answer is B.

**Question 58**

The correct answer is D.

**Question 59**

Not in course anymore.

**Question 60**

The correct answer is D.

**Question 61a**

2.5

**Question 61b**

$$\frac{5}{8}$$

**Question 61c**

1.8

**Question 62a**

0.1881

**Question 62b**

0.2922

**Question 62c**

0.1505

**Question 63**

The correct answer is D.

**Question 64**

The correct answer is D.

**Question 65a**

10

**Question 65b**

8

**Question 66**

The correct answer is C.

**Question 67**

The correct answer is B.

**Question 68a**

$$a = \frac{\pi}{2}$$

**Question 68b**

$$b = 0.5903$$

**Question 68c**

$$c = 0.3634$$

**Question 69**

$$\frac{1}{2} \log_e(3)$$

**Question 70**

The correct answer is E.

**Question 71**

The correct answer is C.

**Question 72a**

$$a = \frac{1}{2}$$

**Question 72b i**

0.7

**Question 72b ii**

$$\frac{3}{5}$$

**Question 73**

The correct answer is C.

**Question 74**

The correct answer is B.

**Question 75a**

0.585

**Question 75b**

0.585

**Question 75c**

0.0156

**Question 75d**

(0.5545, 0.6155)

**Question 75e**

(0.5719, 0.5981)

**Question 100**

$$\text{let } u = x^2 \quad v = \sin x$$

$$u' = 2x \quad v' = \cos x$$

$$\therefore \frac{dy}{dx} = u'v + v'u$$

$$= 2x \sin x + x^2 \cos x$$

**Question 101**

$$f(x) = (x^3 + 3)^{\frac{1}{2}}$$

$$\therefore f'(x) = \frac{1}{2} \times 2x \times (x^3 + 3)^{-\frac{1}{2}}$$

$$= \frac{x}{\sqrt{x^3 + 3}}$$

$$\therefore f'(1) = \frac{1}{\sqrt{4}}$$

$$= \frac{1}{2}$$

**Question 102**

$$\cos(2x) = \frac{\sqrt{3}}{2}$$

$$2x = \frac{\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{11\pi}{12}$$

$$2x \in [0, 20]$$

**Question 103**

$$\log_e(x+2)^2 - \log_e(x) = \log_e(2x+1)$$

$$\log_e \frac{(x+2)^2}{x} = \log_e(2x+1)$$

$$\frac{(x+2)^2}{x} = 2x+1$$

$$(x+2)^2 = 2x^2 + x$$

$$x^2 + 2x + 4 = 2x^2 + x$$

$$0 = x^2 - 3x - 4$$

$$0 = (x-4)(x+1)$$

$$x = -1, 4$$

$$x = 4, \text{ since } x \neq 1$$

**Question 104**

$$2^{3x-3} = 8^{2-x}$$

$$2^{3x-3} = 2^{3(2-x)}$$

$$3x - 3 = 6 - 3x$$

$$6x - 3 = 6$$

$$6x = 9$$

$$x = \frac{3}{2}$$

**Question 105**

$$\int 2 \cos(x) - \sin(2x) dx$$

$$= 2 \sin(x) + \frac{1}{2} \cos 2x + c$$

$$\therefore f(x) = 2 \sin(x) + \frac{1}{2} \cos 2x + c$$

$$f\left(\frac{\pi}{2}\right) = 2 \sin \frac{\pi}{2} + \frac{1}{2} \cos \pi + c = \frac{1}{2}$$

$$\therefore 2 - \frac{1}{2} + c = \frac{1}{2}$$

$$c = -1$$

$$\therefore f(x) = 2 \sin x + \frac{1}{2} \cos 2x - 1$$

**Question 106**

$$y = \frac{k+3-kx}{-3}$$

$$= \frac{1-4x}{k+7}$$

$$\frac{-k}{3} = \frac{4}{k+7}$$

$$-k^2 - 7k = 12$$

$$k^2 + 7k + 12 = 0$$

$$(k+4)(k+3) = 0$$

$$k = -3, -4$$

Check  $k = -3$

$$-3x - 3y = 0$$

$$4x + 4y = 1$$

$\therefore k = -3$  is no solution

Check  $k = -4$

$$-4x - 3y = -1$$

$$4x - 3y = 1$$

$$\therefore k = -4$$

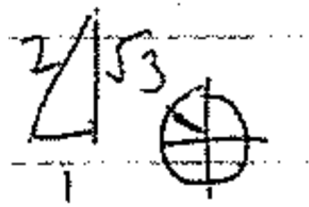
**Question 107**

$$\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2} \quad 2x + \frac{\pi}{3} \in \left[\frac{\pi}{3}, \frac{7\pi}{3}\right]$$

$$2x + \frac{\pi}{3} = \frac{5\pi}{6}, \frac{13\pi}{6}$$

$$2x = \frac{3\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{\pi}{4}, \frac{11\pi}{12}$$

**Question 108**

$$y = -4x + 9$$

$$\therefore \text{mtangent} = \frac{1}{4}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{1}{2\sqrt{x}} = \frac{1}{4}$$

$$2\sqrt{x} = 4$$

$$\sqrt{x} = 2$$

$$x = 4 \therefore y = 2$$

$$2 = -4 \times 4 + a$$

$$a = 18$$

**Question 109**

$$\int_0^9 k\sqrt{x} dx = 27$$

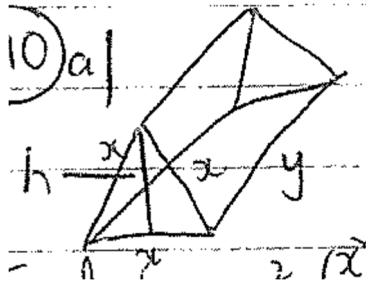
$$\int_0^9 kx^{\frac{1}{2}} dx = 27$$

$$\left[ \frac{2k}{3} x^{\frac{3}{2}} \right]_0^9 = 27$$

$$\frac{2k}{3} \times 9^{\frac{3}{2}} = 27$$

$$\frac{2k}{3} = 1$$

$$k = \frac{3}{2}$$

**Question 10a**

Find h

$$x^2 - \left(\frac{x^2}{2}\right) = h^2$$

$$x^2 - \frac{x^2}{4} = h^2$$

$$\frac{3x^2}{4} = h^2$$

$$h = \frac{\sqrt{3x}}{2}$$

$$\therefore \text{Volume} = \frac{1}{2} \times x \times \frac{\sqrt{3x}}{2} \times y = 1000$$

$$\frac{\sqrt{3x}}{4} y = 1000$$

$$y = \frac{4000}{\sqrt{3x}}$$

**Question 10b**

$$A = 3yx + \frac{1}{2} \times 2 \times \frac{\sqrt{3x}}{2} \times x$$

$$= 3 \times \frac{4000x}{\sqrt{3x^2}} + \frac{\sqrt{3}x^2}{2}$$

$$= \frac{4000\sqrt{3}}{x^2} + \frac{\sqrt{3}x^2}{2} \text{ as req.}$$

**Question 10c**

$$A = 4000\sqrt{3}x^{-1} + \frac{\sqrt{3}}{2}x^2$$

$$\frac{dA}{dx} = -4000\sqrt{3}x^{-2} + \sqrt{3}x$$



$$= \frac{-4000\sqrt{3}}{x^2} + \sqrt{3}x$$

For min let  $\frac{dA}{dx} = 0$

$$\therefore 0 = \frac{-4000\sqrt{3}}{x^2} + \sqrt{3}x$$

$$0 = -4000\sqrt{3}x^3$$

$$4000\sqrt{3} = \sqrt{3}x^3$$

$$x^3 = 4000$$

$$x = \sqrt[3]{4000}$$

### Question 111a

$(2,4) \therefore 4 = 4a + 2t$  – equation 1

$$\frac{dy}{dx} = 2ax + b$$

$$\text{gradient of target} = \frac{0 - 4}{6 - 2}$$

$$= -1$$

$$\therefore -1 = 4a + b$$
 – equation 2

Solving equation 1 and equation 2

$$5 = b \therefore -1 = 4a + 5$$

$$a = \frac{-3}{2}$$

$$a = \frac{-3}{2}, b = 5$$

### Question 111b i

$$\frac{v - 4}{0 - 2} = \frac{0 - 4}{u - 2}$$

$$\frac{4 - v}{2} = \frac{-u}{u - 2}$$

$$4 - v = \frac{-8}{u - 2}$$

$$v = 4 + \frac{8}{u - 2}$$

### Question 111b ii

$$A = \frac{1}{2} \times u \times v - 8$$

$$= \frac{u}{2} \times \left(4 + \frac{8}{u - 2}\right) - 8$$

$$A = 2u + \frac{4u}{u-2} - 8$$

$$\therefore \frac{dA}{du} = 2 - \frac{8}{(u-2)^2}$$

For max/min let  $\frac{dA}{du} = 0$

$$\therefore 0 = 2 - \frac{8}{(u-2)^2}$$

$$\frac{8}{(u-2)^2} = 2$$

$$4 = (2-u)^2$$

$$0 = 4 - 4u + u^2 - 4$$

$$0 = u^2 - 4u$$

$$0 = v(u-4)$$

$$v = 0, 4$$

$$\therefore v = 4 \text{ since } v \neq 0$$

$$\therefore A = 8$$

*Test end points*

$$u = \frac{5}{2} \quad A = 17$$

$$u = 6 \quad A = 10$$

$$\therefore u = 4, A_{\min} = 8 \text{ sq units}$$

#### **Question 111c**

$$u = \frac{5}{2}, A = 17 \text{ sq units}$$

#### **Question 112**

The correct answer is D.

#### **Question 113**

The correct answer is B.

#### **Question 114**

The correct answer is A.

#### **Question 115**

The correct answer is B.

#### **Question 116**

The correct answer is D.

**Question 117**

The correct answer is B.

**Question 118**

The correct answer is E.

**Question 119**

The correct answer is C.

**Question 120**

The correct answer is C.

**Question 121**

The correct answer is E.

**Question 122**

The correct answer is C.

**Question 123**

The correct answer is A.

**Question 124**

The correct answer is A.

**Question 125**

The correct answer is C.

**Question 126**

The correct answer is B.

**Question 27**

The correct answer is E.

**Past Exam Questions**

*2013 Exam 2 Q1*

**Question 1a**

$Max = 27^{\circ}C \quad t = 0, 16h$

**Question 1b**

16 hours

**Question 1c**

$$25 + 2 \cos\left(\frac{\pi t}{8}\right) = 26$$

$$\therefore t = \frac{8}{3}h$$

**Question 1d**

$$T = 26 \quad t = \frac{8}{3}, \frac{40}{3}, \frac{56}{3}$$

$$\therefore \frac{8}{3} + \frac{56}{3} - \frac{40}{3} = 8 \text{ hours}$$

**Question 1e i**

$$\cos\left(\frac{2\pi}{3}\right) = \frac{-1}{2}$$

**Question 1e ii**

'Show that' question

$$\frac{-1}{2} = \frac{0 - \frac{\sqrt{3}}{2}}{c - \frac{2\pi}{3}}$$

$$\frac{-1}{2} = \frac{\frac{-\sqrt{3}}{2}}{\frac{3c - 2\pi}{3}}$$

$$-\frac{1}{2} = \frac{-\sqrt{3}}{2} \times \frac{3}{3c - 2\pi}$$

$$\frac{1}{2} = \frac{3\sqrt{3}}{6c - 4\pi}$$

$$6c - 4\pi = 6\sqrt{3}$$

$$6c = 6\sqrt{3} + 4\pi$$

$$c = \sqrt{3} + \frac{2\pi}{3} \text{ as req.}$$

**Question 1f i**

$$\frac{\sqrt{3}}{2}k = 10 \quad \therefore k = \frac{20\sqrt{3}}{3}$$

$$\sqrt{3}m = 30 \quad \therefore m = 10\sqrt{3}$$

**Question 1f ii**

$$\left(\frac{20\sqrt{3}\pi}{3}, 10\right)$$

2008 Exam 2 Q1

**Question 1a i**

$$0.8^8 = 0.1678$$

**Question 1a ii**

$x = \text{goal}$

$$x - B_i = (8, 0.8)$$

$$\Pr(x - 6) = 0.2936$$

**Question 1b i**

$$0.84^7 = 0.2951$$

**Question 1b ii**

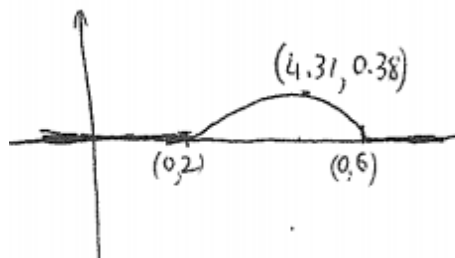
$$GGN \ 0.84 \times 0.84 \times 0.16 = 0.1129$$

$$GNG \ 0.84 \times 0.16 \times 0.64 = 0.0860$$

$$NGG \ 0.16 \times 0.64 \times 0.84 = \frac{0.0860}{0.2849}$$

**Question 1b iii**

Not in course anymore.

**Question 1c i****Question 1c ii**

$$\int_2^3 f(x) dx = 0.1211$$

**Question 1c iii**

$$\int_2^3 xf(x) dx = 4.1333$$

2012 Exam 2 Q3

**Question 3a i**

$$\frac{1}{64}$$

**Question 3a ii**

$$x \sim B_i\left(20, \frac{1}{4}\right)$$

$$\Pr(x \geq 10) = 0.0139$$

**Question 3a iii**

$$n \times \frac{1}{4} \times \frac{3}{4} = \frac{75}{16} \therefore n = 25$$

**Question 3b i**

$$CCCC \ \frac{1}{3} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{64}$$

$$ICCC \ \frac{2}{3} \times \frac{1}{3} \times \frac{3}{4} \times \frac{3}{4} = \frac{1}{8}$$

$$\frac{9}{64} + \frac{1}{8} = \frac{17}{64}$$

**Question 3b ii**

Not in course anymore.

**Question 3c**

$$\Pr(Y > 23) = 6 \Pr(Y = 25)$$

$$\Pr(Y = 25) + \Pr(Y = 24) = 6 \Pr(Y = 25)$$

$${}^{25}C_{25}p(1-p)^0 + {}^{25}C_{24}p^{24}(1-p) = 6 {}^{25}C_{25}p^{25}(1-p)$$

$$\therefore p^{25} + 25p^{24}(1-p) = 6p^{25}$$

$$-5p^{25} + 25p^{24} = 0$$

$$p^{24}(-30p + 25) = 0$$

$$p = 0 \text{ or } \frac{5}{6}$$

$$\therefore p = \frac{5}{6} \text{ since } p \neq 0$$

### Question 3d

$$Y \sim B_i\left(25, \frac{5}{6}\right)$$

$$\Pr(Y \geq 18) = 0.955$$

$$\Pr(Y \geq 22) = 0.381$$

$$*\Pr(z > z) = 0.955$$

$$z = -1.698$$

$$\Pr(z > z_2) = 0.381$$

$$z_2 = 0.301$$

$$\text{using } z = \frac{x - n}{o}$$

$$\therefore -1.698 = \frac{20 - a}{b}, 0.301 = \frac{25 - a}{b}$$

Solving gives

$$a = 24.246, b = 2.500$$

2014 Exam 2 Q4

See Examiners Report for detailed solutions.