Engage Education Foundation

Units 3 and 4 Mathematical Methods (CAS): Exam 1

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Question 1a

$\frac{dy}{dx} = \sin(3x) \times \frac{d}{dx}(\cos(4x)) + \frac{d}{dx}(\sin(3x)) \times \cos(4x)$

$= 3\cos(3x)\cos(4x) - 4\sin(4x)\sin(3x)$

[1] for applying the product rule

[1] for the correct answer

Question 1b

Let $f(x) = \frac{3x^2+1}{\cos(x)}$.

Find $f'\left(\frac{\pi}{3}\right)$.

$f'(x) = \frac{\cos(x) \times \frac{d}{dx}(3x^2+1) - (3x^2+1) \times \frac{d}{dx}(\cos(x))}{\cos^2(x)}$

$= \frac{6x\cos(x) + (3x^2+1)\sin(x)}{\cos^2(x)}$

$\therefore f'\left(\frac{\pi}{3}\right) = \frac{6(\frac{\pi}{3})\cos(\frac{\pi}{3}) + (3(\frac{\pi}{3})^2+1)\sin(\frac{\pi}{3})}{\cos^2(\frac{\pi}{3})}$

$= \frac{2\pi \times \frac{1}{2} + (\frac{\pi^2}{3}+1) \times \frac{\sqrt{3}}{2}}{\frac{1}{4}}$

$= 4(\pi + \frac{\pi^2\sqrt{3}}{6} + \frac{\sqrt{3}}{2})$

[1] for applying the quotient rule

[1] for finding $f'(x)$

[1] for correct answer, or equivalent

Question 2

$\left[\frac{1}{2}\log_e|2x-2|\right]_5^6 = \frac{1}{2}\log_e(10) - \frac{1}{2}\log_e(8)$

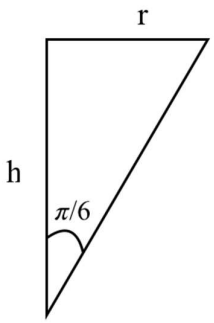
$= \frac{1}{2}\log_e(\frac{5}{4})$

$\therefore p = \frac{5}{4}$

[1] for correctly integrating

[1] for correct answer

Question 3a



$\tan\left(\frac{\pi}{6}\right) = \frac{r}{h} = \frac{1}{\sqrt{3}}$

$h = \sqrt{3}r$

[1] for setting up relevant trigonometric equation

[1] for correct answer

Question 3b

Substituting $h = \sqrt{3}r$ gives:

$V = \frac{\sqrt{3}}{3}\pi r^3 + \frac{2}{3}\pi r^3$

$V = \frac{2+\sqrt{3}}{3}\pi r^3$

[1] for substitution and working

[1] for correct answer

Question 3c

$\frac{dV}{dr} = (2+\sqrt{3})\pi r^2$

[1] for correct answer

Question 4

$2\log_e(4-x) - \log_e(2x-3) = \log_e(3x-2)$

$\therefore \log_e(4-x)^2 - \log_e(2x-3) = \log_e(3x-2)$

$\therefore \log_e\left(\frac{(4-x)^2}{2x-3}\right) = \log_e(3x-2)$

$\therefore \frac{(4-x)^2}{2x-3} = 3x-2$

$\therefore (x-4)^2 = (3x-2)(2x-3)$

$\therefore x^2 - 8x + 16 = 6x^2 - 13x + 6$

$\therefore 5x^2 - 5x - 10 = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore (x - 2)(x + 1) = 0$

$\therefore x = -1$ or $x = 2$

$\therefore x = 2$, since a log can only accept positive values

[1] for writing equation in terms of logarithms

[1] for correct answer

Question 5a

$f'(x) = 2x \log_e(x) + x$

[1] for correct answer

Question 5b

$f'(x) = 2x \log_e(x) + x$

$\therefore 2x \log_e(x) = f'(x) - x$

$\therefore x \log_e(x) = \frac{1}{2}(f'(x) - x)$

$$\therefore \int_{e^{-1}}^{e^4} x \log_e(x)\, dx = \frac{1}{2} \int_{e^{-1}}^{e^4} (f'(x) - x)\, dx$$

$$= \frac{1}{2}\left[f(x) - \frac{x^2}{2}\right]_{e^{-1}}^{e^4}$$

$$= \frac{1}{2}\left[x^2 \log_e(x) - \frac{x^2}{2}\right]_{e^{-1}}^{e^4}$$

$$= \frac{1}{2}\left[4e^8 - \frac{e^8}{2} - (-e^{-2} - \frac{e^{-2}}{2})\right]$$

$$= \frac{1}{2}(\frac{7e^4}{4} + \frac{3}{2e^2})$$

$$= \frac{7e^4}{8} + \frac{3}{4e^2}$$

[1] for correct manipulation

[1] for correct integration

[1] for correct answer

Question 6

$E(\hat{p}) = p = 0.25$

$$sd(\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{\frac{1}{4} \times \frac{3}{4}}{30}} = \frac{1}{4\sqrt{10}}$$

95% confidence interval:

$$p - 2\sqrt{\frac{pq}{n}} < \hat{p} < p + 2\sqrt{\frac{pq}{n}}$$

$$\frac{1}{4} - \frac{1}{2\sqrt{10}} < \hat{p} < \frac{1}{4} + \frac{1}{2\sqrt{10}}$$

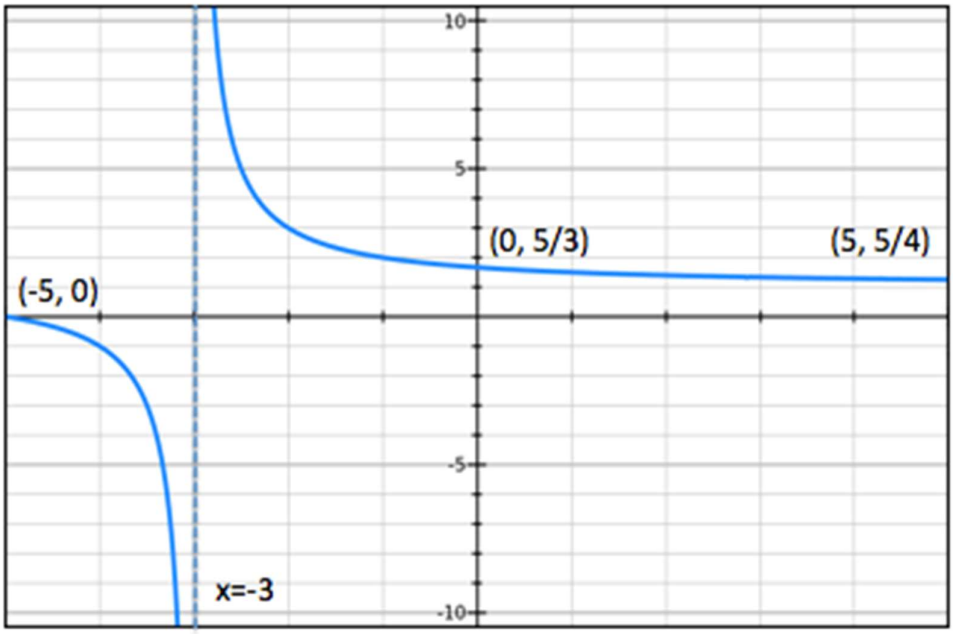
or $\frac{10-2\sqrt{10}}{40} < \hat{p} < \frac{10+2\sqrt{10}}{40}$

[1] for determining the standard deviation

[1] for setting up confidence interval equation with $z = 2$

[1] for correct answer

Question 7a



[1] for correct shape

[1] for correct coordinates of endpoints

[1] for correct coordinates of axis intercepts

Question 7b i

Find the coordinates of the image of the point $(5, \frac{5}{4})$ under a dilation of factor $\frac{1}{2}$ from the y-axis, followed by a translation of 2 units in the positive direction of the y-axis.

$$(x, y) \rightarrow (\frac{1}{2}x, y) \rightarrow (\frac{1}{2}x, y + 2)$$

$\therefore (5, \frac{5}{4}) \rightarrow (\frac{5}{2}, \frac{13}{4})$

[1] for correct answer

Question 7b ii
Given the transformations, we need to find $f(2x) + 2$

$f(2x) + 2 = 3 + \frac{2}{2x + 3}$

[1] for correct functional notation

[1] for correct answer

Question 8a
$E(X) = 0 \times 0.4 + 1 \times 0.2 + 2 \times 0.2 + 3 \times 0.2$
$= \frac{6}{5}$

[1] for correct answer

Question 8b
$Var(X) = E(X^2) - [E(X)]^2$
$= 0 \times 0.4 + 1 \times 0.2 + 4 \times 0.2 + 9 \times 0.2 - (\frac{6}{5})^2$
$= 0.2 + 0.8 + 1.8 - 1.44$
$= 1.36$

[1] for applying computational formula

[1] for correct answer

Question 8c
Pr(X is the same two days in a row)

$= (\Pr(X = 0))^2 + (\Pr(X = 1))^2 + (\Pr(X = 2))^2 + (\Pr(X = 3))^2$
$= 0.4^2 + 0.2^2 + 0.2^2 + 0.2^2$
$= \frac{7}{25}$

[1] for correct interpretation of the question

[1] for correct answer

Question 8d
$\Pr(X \leq 2 | X \geq 1) = \frac{\Pr(1 \leq X \leq 2)}{\Pr(X \geq 1)}$
$= \frac{\Pr(X = 1) + \Pr(X = 2)}{\Pr(X = 1) + \Pr(X = 2) + \Pr(X = 3)}$
$= \frac{0.4}{0.6}$

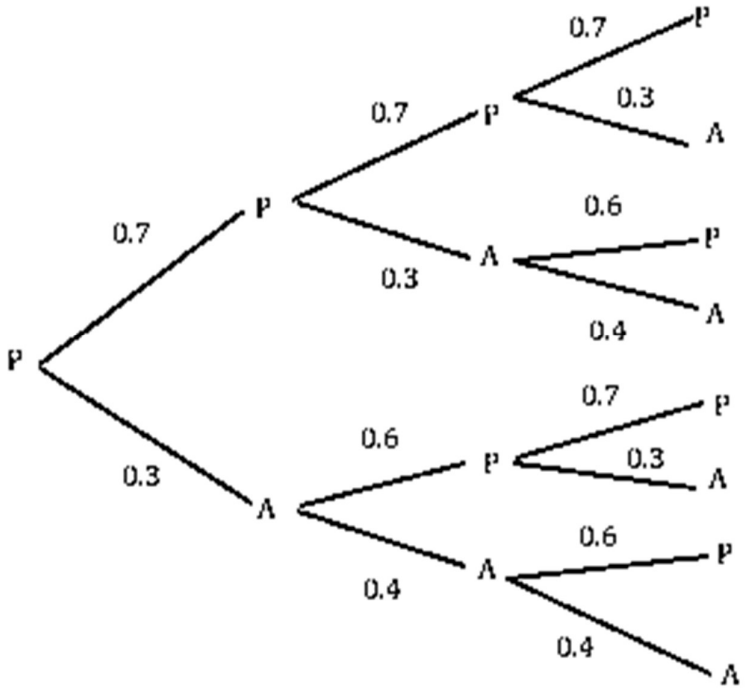
$= \frac{2}{3}$

[1] for correct use of conditional probability formula

[1] for correct answer

Question **9**
Let P be the event that he goes to the Pollos.

Let A be the event that he goes to Altiplano.



$\Pr(\text{goes to Altiplano on exactly one of next three Saturdays}) = \Pr(\text{PAP}) + \Pr(\text{PPA}) + \Pr(\text{APP})$

$= 0.7 \times 0.3 \times 0.6 + 0.7^2 \times 0.3 + 0.3 \times 0.6 \times 0.7$

$= 0.399$

[1] for drawing correct tree diagram

[1] for correct identification of relevant probabilities

[1] for correct answer

Question **10**

Since the curve is below the x-axis, we take the signed area.

Area of region $= -\int_C^0(-2e^{-3x-5}-2)dx = \int_C^0(2e^{-3x-5}+2)dx = -\frac{2}{3e^5}+\frac{2}{3e}+\frac{8}{3}$

$\therefore \left[2x-\frac{2}{3}e^{-3x-5}\right]_C^0 = -\frac{2}{3e^5}+\frac{2}{3e}+\frac{8}{3}$

$\therefore -\frac{2}{3e^5}-2C+\frac{2}{3}e^{-3C-5} = -\frac{2}{3e^5}+\frac{2}{3e}+\frac{8}{3}$

By inspection, $C = -\frac{4}{3}$

[1] for initially writing equation in terms of integrals

[1] for writing equation without integrals

[1] for correct answer