Engage Education End of Year Revision Lectures
Further Maths Matrices Solutions

1) C – 3x2
2) D
3) E – 7

$\begin{bmatrix} 3 & 2 \\ 6 & x \end{bmatrix}$

$det = 3x - 2(6)$

$\therefore 9 = 3x - 12$

$\therefore 21 = 3x$

$\therefore 7 = x$ E.

4) B
5) D – 10
6) D – BC – the number columns of the 1st matrix must match the number of rows of the 2nd matrix.
7) E – (3x3)
8) B = BA – 2A

A. $BA^2 \rightarrow$ cannot find A^2 as A is not square

B. $BA - 2A$

$BA \rightarrow (3\times3)(3\times4)$, $2A \rightarrow (3\times4)$

$(3\times4) \rightarrow$ same order so can simplify this matrix expression.

$\boxed{B}$

C. Different orders, can't add

D. AB cannot be calculated

E. A^{-1} does not exist A not square

9) B
10) Cannot divide by matrices so pre-multiply both sides by A^{-1}

$\therefore x = A^{-1}\begin{bmatrix} 5 & 6 \\ 8 & 10 \end{bmatrix}$

$x = \begin{bmatrix} 8 & 4 \\ 5 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 5 & 6 \\ 8 & 10 \end{bmatrix}$

$x = \begin{bmatrix} -4.25 & -5.5 \\ 9.75 & 12.5 \end{bmatrix}$

B.

11) B
12) a.

$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

b) $\begin{bmatrix} p \\ r \\ a \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 33 \\ 40 \\ 43 \end{bmatrix}$

$\therefore x = -5 \quad = \begin{bmatrix} 7 & -1 & -4 \\ -1 & 0 & 1 \\ \boxed{-5} & 1 & 3 \end{bmatrix} \begin{bmatrix} 33 \\ 40 \\ 43 \end{bmatrix}$

c. Oscar is predicted to have 10 rebounds

13) C – 3

$\begin{bmatrix} -4 & 2 \\ 2 & 1 \end{bmatrix}$ Det A (4-4) = 0 ×

$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = 1$ B (tick)

$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = 2$ C (tick)

$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} = 0$ ×

The pair that gives a single x and y value as a solution has a unique solution.

$\left.\begin{matrix} 4x+2y=10 \\ 2x+y=5 \end{matrix}\right\} \rightarrow$ Multiples of each other $\therefore$ Same line

$\left.\begin{matrix} x=0 \\ x+y=6 \end{matrix}\right\} \rightarrow x=0,\ y=6$

$\left.\begin{matrix} x-y=3 \\ x+y=3 \end{matrix}\right\} \rightarrow x=3,\ y=0$

$\left.\begin{matrix} 2x+y=5 \\ 2x+y=10 \end{matrix}\right\} \rightarrow$ Inconsistent $\therefore$ Parallel lines (same gradient)

$\left.\begin{matrix} x=8 \\ y=2 \end{matrix}\right\} \rightarrow x=8,\ y=2$

$\boxed{C}$

$\therefore$ 3 sets have a unique sol$^{\underline{n}}$.

$T = D + D^2$

14) a. The figures in bold are in the diagonal, therefore represent the fact that the musicians cannot compete against themselves

15) c. George beat Keith who beat Ian

d. Keith beat Ian who beat Josie & Keith beat Harriet who beat Josie so x=2

e. First – Keith, Last – Ian

16) a. A and B

b. To figure this out, the matrix should be squared, which will show 2 step communication. Any communication channels that were present in the 1 step matrix are redundant. BA, DA, BD are all redundant – therefore the answer is 3.

$$\begin{array}{c c} \begin{array}{cccc} A & B & C & D \end{array} & \\ \begin{bmatrix} \mathbf{2} & 1 & 1 & 1 \\ 1 & \mathbf{3} & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & \mathbf{2} \end{bmatrix} & \begin{array}{c} A \\ B \\ C \\ D \end{array} \end{array}$$

17) a.

$$\begin{vmatrix} 0.8 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{vmatrix}$$

b.
300 000
120 000
180 000

18) a. i.

$$S_2 = TS_1$$
$$= \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} 540 \\ 36 \end{bmatrix} = \begin{bmatrix} 493.2 \\ 82.8 \end{bmatrix} \quad \boxed{S_2 = \begin{bmatrix} 493 \\ 83 \end{bmatrix}}$$

ii.

$$S_5 = T^4 S_1$$
$$= \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}^4 \begin{bmatrix} 540 \\ 36 \end{bmatrix} = \begin{bmatrix} 421.46 \\ 154.54 \end{bmatrix}$$

$\boxed{\therefore \text{ 421 Attend 5th History lecture}}$

19) a. i.

$$A_2 = GA_1 = \begin{bmatrix} 1.2 & -0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} 2000 \\ 1000 \end{bmatrix} \boxed{= \begin{bmatrix} 2100 \\ 1100 \end{bmatrix} \begin{array}{c} D \\ S \end{array}}$$

ii.

$$2100 + 1100 = 3200$$

b.

$$A_{10} = G^9 A_1 = \begin{bmatrix} 1.2 & -0.3 \\ 0.2 & 0.7 \end{bmatrix}^9 \begin{bmatrix} 2000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 2612.6 \\ 1612.6 \end{bmatrix} \simeq \begin{bmatrix} 2613 \\ 1613 \end{bmatrix}$$

c. Attendance at Dinosaurs games is expected to increase to 3000 people at which point it will remain steady.

Steady State

Game 80

$$A_{80} = G^{79} A_1 = \begin{bmatrix} 2999.8 \\ 1999.8 \end{bmatrix} \simeq \begin{array}{c} \begin{bmatrix} 3000 \\ 2000 \end{bmatrix} \end{array}\begin{array}{c} D \\ S \end{array}$$

Game 81

$$A_{81} = G^{80} A_1 \simeq \begin{bmatrix} 3000 \\ 2000 \end{bmatrix}\begin{array}{c} D \\ S \end{array}$$

d. Attendance is expected to decrease to 600 at which point it will remain steady.

$$A_1 = \begin{bmatrix} 2000 \\ 1800 \end{bmatrix}$$

$$A_{80} = G^{79} A_1 \simeq \begin{bmatrix} 600 \\ 400 \end{bmatrix}\begin{array}{c} D \\ S \end{array}$$

$$A_{81} = G^{80} A_1 \simeq \begin{bmatrix} 600 \\ 400 \end{bmatrix}\begin{array}{c} D \\ S \end{array}$$

20) a. 68 singing rehearsals

$$L_2 = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix}\begin{bmatrix} 95 \\ 97 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 100 \\ 80 \end{bmatrix} \quad L_3 = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix}\begin{bmatrix} 100 \\ 80 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 100 \\ 68 \end{bmatrix}\begin{array}{c} D \\ S \end{array}$$

b. 12 students will no longer turn up to rehearsal

Subtracted column $\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

21) a. 1x5

b. i.

$$R = \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 23 & 57.5 & 80.5 & 207 & 92 \\ 18 & 45 & 63 & 162 & 72 \end{bmatrix} \end{array}\begin{array}{c} B \\ C \end{array}$$

ii. The number of students who received a D for chemistry

c. i.

$$F = \begin{array}{c} \begin{array}{cc} B & C \end{array} \\ \begin{bmatrix} 110 & 95 \end{bmatrix} \end{array}$$

ii. $84800 fees paid

$$L = F\,N \qquad F: (1\times2) \quad N: (2\times1)$$

$$L = \begin{bmatrix} 110 & 95 \end{bmatrix}\begin{bmatrix} 460 \\ 360 \end{bmatrix}\begin{array}{c} B \\ C \end{array} \quad L = \begin{bmatrix} 84800 \end{bmatrix}$$

22) a. i.

$$S_2 = TS_1 = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} 540 \\ 36 \end{bmatrix}$$
$$= \begin{bmatrix} 493.2 \\ 82.8 \end{bmatrix} \simeq \begin{bmatrix} 493 \\ 83 \end{bmatrix}$$

ii. 421 students attended the History lecture.

$$S_5 = T^4 S_1 = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}^4 \begin{bmatrix} 540 \\ 36 \end{bmatrix}$$
$$= \begin{bmatrix} 421.46 \\ 154.54 \end{bmatrix} \simeq \begin{array}{c} \begin{bmatrix} 421 \\ 155 \end{bmatrix} \end{array} \begin{array}{l} A \\ N \end{array}$$

b.
$S_n = T^{n-1} \quad S_1$

c. Lecture 8

d. 348

23) a.

$O_{2009} = A\,S_{2008} + BA = \pi r^2$

$$= \begin{bmatrix} 0.75 & 0 \\ 0 & 0.68 \end{bmatrix} \begin{bmatrix} 456 \\ 350 \end{bmatrix} + \begin{bmatrix} 18 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} 360 \\ 250 \end{bmatrix}$$

b.

$$O_{2009} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix} \begin{bmatrix} 500 \\ 360 \end{bmatrix} - \begin{bmatrix} 40 \\ 38 \end{bmatrix} = \begin{bmatrix} 360 \\ 250 \end{bmatrix}$$

$$O_{2010} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix} \begin{bmatrix} 360 \\ 250 \end{bmatrix} - \begin{bmatrix} 40 \\ 38 \end{bmatrix} = \begin{bmatrix} 248 \\ 162 \end{bmatrix} \begin{array}{l} M \\ P \end{array}$$

$\boxed{\therefore 248 \text{ Maths Books}}$

24)

$$S_{2009} = \begin{bmatrix} 0.88 & 0.52 & 0.65 \\ 0.10 & 0.44 & 0.10 \\ 0.02 & 0.04 & 0.25 \end{bmatrix}^2 \begin{bmatrix} 880 \\ 230 \\ 120 \end{bmatrix} = \begin{bmatrix} 996.9 \\ 191.4 \\ 41.7 \end{bmatrix} \begin{array}{l} P \\ F \\ D \end{array}$$

$\boxed{\therefore 42 \text{ will defer!}}$