

Engage Education End of Year Revision Lectures
Further Maths Core Solutions

Core – Data

- 1) D – temperature is numerical, town is categorical
- 2)

(VCAA Exam 2 2006)

② Displaying Univariate Data

STATISTICS App on calculator

1 Variable Statistics

5 Figure Summary

$n = 14$

$\min = 76$

$Q_1 = 80$

$\text{med} = 83$

$Q_3 = 85.8$

$\text{max} = 89.8$

Draw boxplot scaled accurately!

- 3) Walked – 10; Sat or Stood – 7; Ran – 8
- 4) Addition of 16 to the stem and leaf plot

stem (10s)	leaf (units)
0	0
1	2 4 6
2	0 1 3 3 4 4 4
3	2 3 3 6 7 8
4	3 4 7 8
5	6

- 5) A – 14%, from the graph
- 6) Yes, an association is indicated. % who ran decreased as year level increased i.e. 78%
-> 40% -> 10%
- 7) E – 86%, from the bar graph
- 8) A – 1, as 1 has the most dots in it

$$\frac{7 + 1}{2} = 8\text{th position}$$

- 9) 580 – there are 3 x 580 in the stem and leaf plot
- 10) B – 6. Use a calculator and 1VarStats to get a 5 figure summary for the data. Then calculate the IQR by hand.
 $Q_1 = 5^{\text{th}} \text{ score} = 19$
 $Q_3 = 14^{\text{th}} \text{ score} = 25$
 $\text{IQR} = 25 - 19 = 6$
- 11) C – 20%. Add the total number of countries and divide by 2, then see what range that fits into.

Sum = 54

$$\frac{54 + 1}{2} = \frac{55}{2} = 27.5$$

- 12) B – 40 and 45
(VCAA Exam 1 2006)

Need to cross over halfway mark of 50% from the graph

$$2+3+2+3+6+6+7+11+14=54$$

$54/2 = 27$, therefore the 27th value is the median. This falls between 40-45.

13) C - 25%

Greater than 14mm is the upper quartile - C

$\therefore Q_3 = 14\text{mm}$ which gives 25% percentage

14) E - process of elimination

15) C - look at the range to determine whether they're more variable, and notice that 75% of jellyfish from location A are less than 14mm diameter whilst 75% of jellyfish in location B are greater than 15mm in diameter. Therefore they are smaller overall at location A.

16) Yes it is an outlier.

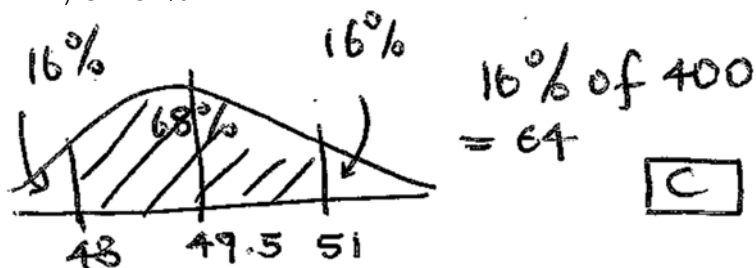
From boxplot $Q_1 = 160$ $Q_3 = 170$

$$IQR = 170 - 160 = 10$$

$$\begin{aligned} \text{Lower Cutoff} &= Q_1 - 1.5IQR \\ &= 160 - 1.5 \times 10 \\ &= 160 - 15 \\ &= 145 \end{aligned}$$

New value 140 is still less than 145 (lower fence/cutoff) so is still an outlier.

17) C - 64%



18) E - 125.9 and 6.0

Use statlist editor on calculator.

$$\begin{aligned} 1 \text{ Var stats } \bar{x} &= 125.93 \dots \\ \text{mean} &\rightarrow \bar{x} \approx 125.9 \text{ (1 dp)} \\ \text{st. dev} &\rightarrow s_x = 5.957 \dots \\ & s_x \approx 6.0 \text{ (1 dp)} \end{aligned}$$

Caful: DO NOT USE σ_x !

19) The distribution is approximately symmetrical therefore both mean and median are appropriate measures of centre and spread

20) -1.4 \therefore below average

$$z = \frac{\text{Score} - \text{Mean}}{\text{st. dev}}$$

$$= \frac{(83.1 - 89.3)}{4.5}$$

$$= -1.4$$

Note:
 * Remember to include brackets when calculating z value on calculator.

21) Yes

Yes, age of women at first marriage has increased over the years.

ie 30 years and over, % has increased from 26% in 1986 to 33.3% in 1996 to 42% in 2006

22) Positive related

Positively related means as one increases, the other tends to increase.

The boxplots show that as age of boys increases, median height of boys increases also.
 ie 15mth (83cm) to 27mth (89.5cm) to 36mth (95cm)

23) a. male income

b) \$0.35 for \$1 increase
 $\therefore \$0.35 \times 1000$ for \$1000 increase
 $= \$350$

24) i. $FI = 1300 + 0.35 \times 15000 = 18250$

ii. This prediction is using extrapolation by going outside of the data range, therefore is unlikely to be accurate.

25) E

24) Step 1 Find 'm'

$$m = \frac{r s_y}{s_x} = \frac{-0.5675 \times 6.98}{2.61}$$

$$\therefore m = -1.5177$$

Step 2 Find 'c'

$$c = \bar{y} - m\bar{x}$$

$$= 23.93 - (-1.5177)(4.56)$$

$$\therefore c = 30.85$$

$$\therefore y = 30.85 - 1.5177x$$

$$y \approx 30.9 - 1.52x \quad \boxed{A}$$

26) As the residual plot appears random, the relationship is likely to be linear

27) a. 0.53

b) $r^2 = 0.7541^2 = 0.6$

c) 60% of the variation in height can be explained by variation in age

28) E – an increasing trend with seasonal variation

29) E – an increasing trend only

30) D – 446 – the month number will be 36. Substitute into equation.

3 years x 12 months = 36 months

Sub into equation = 446

31) C – 70

32) C – 360

4 point moving mean with
centring in month no. 5

3 4 5 6 4 point
354 356 373 353 AV = 359

4 5 6 7 4 point
356 373 353 364 AV = 361.5

Centred 4 point $\frac{359 + 361.5}{2}$
 ≈ 360.25
 ≈ 360 C

33) D – 375

34) D – 0.98 – all the seasonal indices should add up to 12 (the number of values there are)

35) C – 351 – deseasonalised value = actual value/seasonal index

36) A – 304 – use the number 6 as month number, then multiply it by the seasonal index

37) a. 1.1

b. 241mm

$$\text{Des} = \frac{\text{Actual}}{\text{SI}} = \frac{188}{0.78} = 241.03 \approx 241 \text{ m}$$

c. rainfall in autumn is typically 5% above average

Core – Recursion and Financial Modelling

1) B – 200

2) C – \$56250

3) a. $V_{n+1} = V_n + (3.4/100 \times 18000)$

b. 1 year - $V_1 = V_0 + (3.4/100 \times 18000) = \18612

2 years - $V_2 = V_1 + (3.4/100 \times 18000) = \19224

3 years - $V_3 = V_2 + (3.4/100 \times 18000) = \19836

c. $V_n = V_0 + nD$

$22,000 = 18000 + n \times (3.4/100 \times 18000)$

$n = 6.5 \text{ years} = 7 \text{ years}$

4) C – $V_0 = N$, $V_{n+1} = V_n + (3N/100)$

5) D – \$20,000

6) E – \$113000

7) D - \$178

8) C - 12%

\$2000 - \$200 = \$1800 Borrowed

\$68 x 36 = \$2448 Repay

\$2448 - \$1800 = \$648
Interest Paid

$$SI = \frac{Prt}{100}$$

$$648 = \frac{1800 \times r \times 3}{100}$$

$$r = 12$$

C

9) D - \$27.90

\$720 - \$180 = \$540
Borrow

$$SI = \frac{\$540 \times 12 \times \frac{5}{2}}{100} = \$129.60$$

$$\text{Ave} = \$540 + \$129.60 \\ = \$669.60$$

$$\text{Monthly Repayment} = \frac{\$669.60}{24} = \$27.90$$

10) B - 16.7%

\$500 - \$50 = \$450 Borrow

Pay back \$25 x 2 x 12 = \$600

Interest \$600 - \$450
= \$150

$$150 = \frac{P \times r \times t}{100}$$

$$\therefore r = 16.67\%$$

11) D - \$24392

12) D - 10000 x 1.05¹⁰

13) C - 8000 x 1.018¹⁶ - 8000 x 1.018¹²

14) a. $V_n = 7600 \times (1 + [(6/12)/100])^n$

b. $V_n = 7600 \times (1 + [(6/12)/100])^n$

$$V_3 = 7600 \times (1 + [(6/12)/100])^3 = \$7714.57$$

15) a. Option A = $r = [(1 + (8.3/12)/100)^{12} - 1] \times 100\% = 8.621\%$

Option B = $r = [(1 + (7.8/52)/100)^{52} - 1] \times 100\% = 8.104\%$

b. Option A: $V_1 = 35000 \times (1 + [(8.3/12)/100])^{12} = \38018

Option B: $V_1 = 35000 \times (1 + [(7.8/52)/100])^{52} = \37837

c. Option B, as it will compound less interest given the lower effective interest rate

16) D - \$7132.42

$$N = 20 \times 4 = 80 \text{ payments}$$

$$I = 7.25$$

$$FV = 0$$

$$PV = 300000$$

$$P/Y = 4$$

$$C/Y = 4$$

$$\boxed{PMT} = \boxed{-7132.42}$$

d

$$17) B - \$413$$

$$N = 4 \times 12 = 48 \text{ payments}$$

$$I = 6.25$$

$$FV = 0$$

$$PV = 17500$$

$$P/Y = 12$$

$$C/Y = 12$$

$$\boxed{PMT} = -412.99$$

$$18) A - 6.3\%$$

$$N = 7 \times 4 = 28$$

$$I = \boxed{6.3}$$

$$PV = 100000$$

$$FV = -80000$$

$$PMT = -2150$$

$$P/Y = 4$$

$$C/Y = 4$$

$$19) B - \$2457.60$$

$$N = 5 \times 12 = 60$$

$$I = 9.2$$

$$PV = 18000$$

$$FV = 0$$

$$PMT = \boxed{-375.40}$$

$$P/Y = 12$$

$$C/Y = 12$$

After 10 repayments

$$N = 10$$

$$PV = 18000$$

$$FV = \boxed{-15542.40}$$

$$PMT = -375.40$$

$$P/Y = 12$$

$$C/Y = 12$$

$$\text{Paid off } \$18000 - \$15542.40$$

$$= \$2457.60$$

$$20) D - \$5711$$

$$\begin{aligned}
N &= 2 \\
I\% &= 5 \\
PV &= -4000 \\
PMT &= -800 \\
FV &= \boxed{5710.625} \\
P/Y &= 4 \\
C/Y &= 4
\end{aligned}$$

- 21) B - 15%
- 22) a. i. 10% of 60000 = \$6000
ii. $6000 - 3 \times 6000 = \$42000$
iii. $6000 - 6000n = 12000$
 $6000n = 48000$
 $N = 8$
b. i. 15%
ii. $V = 60000 \times (0.85)^3 = \36847.3
iii. $6000 \times 0.85^n < 12000$
 $n = 9.9$
- 23) a. $V_{n+1} = (1 - (4/100)) \times V_n$, $V_0 = 9800$
b. Use the formula to find the values
9408, 9031.68, 8670.41, 8323.59
c. From the sequence above, $V_3 = 8670.41$ and $V_4 = 8323.59$.
Therefore, the depreciation of the motorcycle in the 3rd year = $V_4 - V_3 = \$346.82$
- 24) E - \$35828
- 25) a. \$720
b. simple interest, as the value increases by the same amount each year
c. \$96
d. 13.3%
- 26) a. $7.5/4 = 1.875\%$
b. \$375
c. \$3926
d. \$4227
e. \$1139
- 27) B - \$75
- 28) a. Using finance solver - 35.99 periods = 36 periods = 3 years
b. Using finance solver find the value of the loan after the 19th payment
N: 19
I: 3.6
PV: 150000
Pmt: -4896.66
FV: ? = -76866.48
Ppy: 4
Cpy: 4
Using finance solver, find the value of the loan after the 20th payment.
N: 1
I: 3.6
PV: 76866.48
Pmt: -4896.66
FV: ? = -72661.62
Ppy: 4
Cpy: 4

Subtract FV (20) from FV (19) to find the principal paid in the 20th payment

$$76866.48 - 72661.62 = \mathbf{4204.86}$$

Subtract the principal paid from the payment to find the amount of interest paid in the 20th payment.

$$4896.66 - 4204.86 = \mathbf{\$691.8}$$