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Free Exam
for 2006-15 VCE study design

Engage
Education
Foundation

Units 3 and 4 Further Maths: Exam 2

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Core: Data analysis

Question 1a

$\text{mean} = \frac{17+18+20+20+22+22+25+26+27+30+30+30+31+32+51}{15} = 26.73$ [1]

mode = 30 [1]

median = 26 (when there are 15 data points, the 8th data point in the median) [1]

Question 1b

$Q_3 = 30$

$Q_1 = 20$

IQR = 10

Q_3 + 1.5IQR = 30 + (1.5 x 10) = 45

Q_1 – 1.5IQT = 20 – (1.5 x 10) = 5

51 is an outlier as 51 > 45 [2]

Question 1c

Mother's age is the independent variable as it is plotted on the x axis [1]

Question 1d

The data looks fairly linear. A residual plot for linear data has no pattern, it looks random. [1]

So it is likely that the residual plot for this data would look random. [1]

Question 1e

If the analyst applied a $\frac{1}{x}$ transformation: [1]

r = -0.9494

r^2 = 0.9013

If the analyst applied a $log_{10}x$ transformation: [1]

r = 0.9330

r^2 = 0.8705

It is better to use a $\frac{1}{x}$ transformation because it gives a slightly higher r and r^2 value, indicating a slightly more linear relationship. [1]

Question 1f

father's age = 55 [0.5]

mother's age = 30 [0.5]

z-score = $\frac{30-mean}{standard\ deviation} = \frac{30-26.73}{8.35} = 0.39$ [1]

Question 1g
To compare variability we can compare range, IQR and standard deviation.

Mother's age: [0.5]

- Range = 34
- IQR = 10
- Standard deviation = 8.35

Father's age: [0.5]

- Range = 53
- IQR = 28
- Standard deviation = 15.14

The father's age category is more variable as it has a larger range, IQR and standard deviation. [1]

Module 1: Number patterns

Question 1a
The sequence is geometric. [1]

Question 1b
$I_n = 20 \times 1.1^n$ [1]

Question 1c
$C_n = 20 + 20 \times 1.1^n$ [1]

Question 1d
$C_{24} = 20 + 20 \times 1.1^{24}$

$C_{24} = 20 + 196.99$

$C_{24} = 216.99$

Answer: $216.99 [1]

Question 1e
Account Type B [1]

Question 1f
$(20 \times 1.1^n) > (20 + n \times 30)$

$x > 44.151$

At the end of the 45th month the interest received from Account Type A will exceed the interest received from Account Type B. [1]

Question 2a
$G_n = 5 \times 0.8^n$ [1]

Question 2b
Some theory behind the answer:

- gain between 1st and 2nd visits = $G_1 = 5 \times 0.8 = 4$
- 4 + 5 = 9 kg (weight during 2nd visit)
- therefore between 5th and 6th visit = G_5

$G_5 = 5 \times 0.8^5 = 1.64$g [1]

Question 2c
a = 5 and r = 0.8

$$S_\infty = \frac{a}{1-r} = \frac{5}{1-0.8} = \frac{5}{0.2} = 25 \text{ kg}$$

Total weight gain = 25 kg [1]

Total weight = 25 + 5 (original weight) = 30 kg [1]

Question 3a
$t_n = t_{n-1} \times 0.5 + 32$

$t_{14} = 48$

$t_{15} = 48 \times 0.5 + 32 = 56$

$t_{16} = 56 \times 0.5 + 32 = 60$ [1]

Question 3b
$t_{17} = 60 \times 0.5 + 32 = 62$

$t_{18} = 62 \times 0.5 + 32 = 63$

$t_{19} = 63 \times 0.5 + 32 = 63.5$ [1]

63.5 - 62 = 1.5

Final answer: 2 people [1]

Question 3c
From 12 to 1:30 = 90 minutes = 9 × 10 minutes

$t_9 = 9 \times 4 = 36$ guests arrived

48 - 36 = 12 guests not yet arrived [1]

Question 3d
$t_n = 48 = n \times 4$

n = 12

12 × 10 minutes = 120 minutes

120 minutes from 12pm = 2pm [1]

Module 2: Geometry and trigonometry

Question 1a

Angle BAC = 60° [1]

ABC is an equilateral triangle, so all angles are the same.

Question 1b

Heron's formula: A = $\sqrt{s(s-a)(s-b)(s-c)}$

Because ABC is an equilateral triangle, all sides lengths are the same, so $a = b = c$.

Let x represent the side lengths.

$s = \frac{3x}{2} = 1.5x$ [1]

$A = \sqrt{1.5x(1.5x - x)(1.5x - x)(1.5x - x)}$ [1]

$A = \sqrt{1.5x(0.5x)(0.5x)(0.5x)}$

$A = \sqrt{0.1875x^4}$ [1]

We know that A = 1cm^2

$1 = \sqrt{0.1875x^4}$

$1 = 0.1875x^4$

$x^4 = \frac{1}{0.1875}$

$x^4 = 5.333333$

$x = \sqrt[4]{5.3333}$ [1]

$x =$1.40cm

Question 2a

A = $\frac{1}{2}bh$

A = 0.5 x 1 x 1 = 0.5cm^2 [1]

Question 2b

Remaining area = 3 parts

Star = 1 part

So the triangle has a total of 4 parts. [1]

4 parts = 0.5cm^2 [1]

1 part = 0.5/4 = 0.125cm^2 [1]

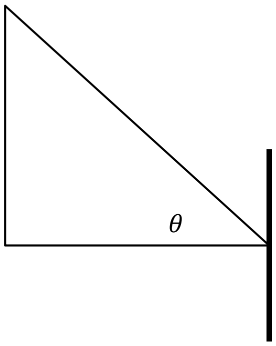
Question 3a

angle of depression = angle of elevation = θ

$tan\theta = \frac{300}{100}$

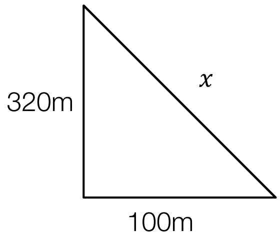
$\theta = tan^{-1}\frac{300}{100}$ = 71.57° [1]

Question 3b



Bearing = 360 – $\theta = 288.43°$ [1]

Question 3c



$x^2 = 320^2 + 100^2$

$x = 335m$ [1]

Question 4a

$A = \frac{1}{3}\pi r^2 h$

$A = \frac{1}{3} \times \pi \times 1^2 \times 4$

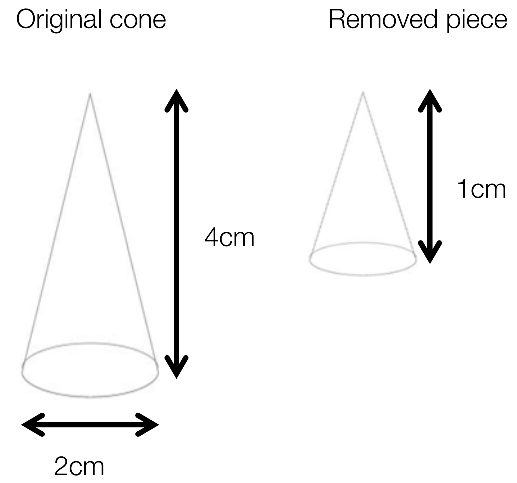
$A = 4.19\ cm^3$ [1]

Question 4b

height of truncated cone = ¾ x 4 = 3cm

height of removed piece = 1 cm

To find the volume of truncated piece, we first need to find volume of removed piece.



Length scale factor = ¼

Volume scale factor = $\frac{1}{4^3} = \frac{1}{64}$

Area of removed cone = $\frac{1}{64} \times area\ of\ orginal\ cone$ = 4.19/64 = 0.0654 [1]

Area of removed cone + area of truncated cone = area of original cone

Area of truncated cone = area of original – area of removed cone

Area of truncated cone = 4.19 – 0.0654 = 4.1246cm^2 [1]

Module 3: Graphs and relations

Question 1a

Break-even when $R = C$

$3.5x = 20 + 2.5x$

$x = 20$ [1]

Question 1b

$x = 0$

$R = 0$

$C = 20$

Profit = -20

Loss = $20 [1]

Question 1c

$R = 4.5x$ [1]

$C = 22 + 2.5x$ [1]

Question 1d

In one day:

$R = 4.5 \times 100 = \$450$

$C = 22 + (2.5\backslash times100) = 22 + 250 = 272$

Profit = $R – C = 450 – 272 = 178$ [1]

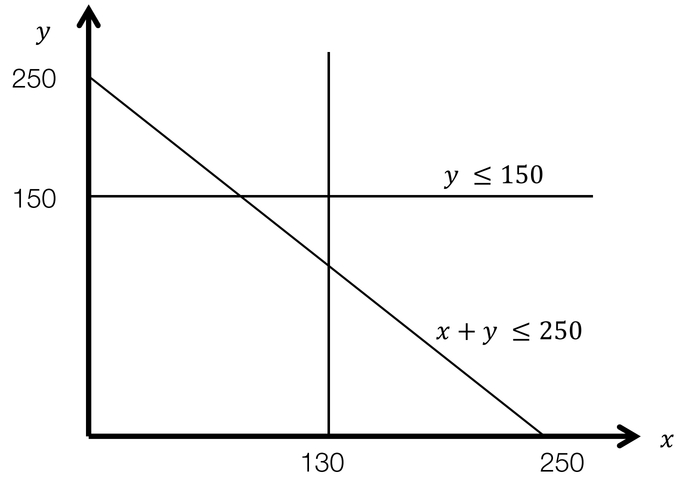
3 weeks = 3 x 7 days = 21 days

Total profit = 178 x 21 = $3738 [1]

Question 2a

The total number of cupcakes cannot exceed 250. [1]

Question 2b



Question 2c

$R = 4.5x + 4y$

Calculate the profit at each corner.

(0, 150)

$R = (4.5 \times 0) + (4 \times 150) = 600$ [0.5]

(130, 0)

$R = (4.5 \times 130) + (4 \times 0) = 585$ [0.5]

(100, 150)

$R = (4.5 \times 100) + (4 \times 150) = 1050$ [0.5]

(130, 120)

$R = (4.5 \times 130) + (4 \times 120) = 1065$ [0.5]

At the revenue maximising point 130 chocolate cupcake and 120 vanilla cupcakes are sold. [1]

Question 3a

$s = \frac{d}{t}$

$t = \frac{d}{s}$

$t = \frac{0.9km}{9km/h} = 0.1\ hours = 6\ minutes$ [1]

Question 3b

$t = \frac{0.9km}{3km/h} = 0.3\ hours = 18\ minutes$ [1]

Total time = 6 +18 = 24 minutes

Therefore, Lea Yen arrives to work at 8.24am – she is not late [1]

Module 4: Business-related mathematics

Question 1a

$I = \frac{PRT}{100}$

$\therefore 2030 = \frac{7000 \times 10 \times T}{100}$

$\therefore 2030 = \frac{7000 \times T}{100}$

$\therefore 203000 = 70000 \times T$

$\therefore \frac{203000}{70000} = T$

$\therefore 2.9 = T$

Therefore, the period is 2.9 years, to 1 decimal place [1]

Question 1b

$\frac{2030}{7000} \times 10 = 29$

$\therefore 29\%$ [1]

Question 1c

$I = PRT$

$\therefore I = \frac{7000 \times 10 \times 2}{100}$

$\therefore I = \$1400$

Question 2a

Total amount paid:

$(25 \times 36) + 100 = 900 + 100 = \$1000$

∴ as Xbox is worth $700, total interest paid:

$1000 - 700 = \$300$

Question 2b

Use flat rate of interest formula:

$rf = \frac{100 \times I}{P \times t}$

$\therefore rf = \frac{100 \times 300}{600 \times 3}$

$\therefore rf = 16.6666666\% \approx 16.7\%$

Therefore, the flat rate of interest is 16.7%, to 1 decimal place [1]

Question 2c
Use the effective interest rate formula:

$$re = \frac{100\ \times I}{P \times t} \times \frac{2 \times n}{(n+1)}$$

$$\therefore re = rf \times \frac{2 \times n}{(n+1)}$$

$$\therefore re = 16.7 \times \frac{2 \times 36}{(36+1)}$$

$$\therefore re = 16.7 \times \frac{72}{37}$$

$$\therefore re = 32.49729$$

$\therefore re = 32\%$ (to nearest percent) [1]

Question 3a
Number of pages printed = $5000 \times 5 = 25000$

Depreciation = $25000 \times 0.06 = \$1500$ [1]

Therefore, value of printer = $20000 - 1500 = \$18500$ [1]

Question 3b
Flat rate depreciation:

$$D = \frac{PRT}{100}$$

$$\therefore D = \frac{20000 \times 1.51 \times 5}{100}$$

$$\therefore D = \$1510$$

Question 3c
After 5 years, value of:

Printer A = $\$18500$(from part a)

Printer B = $20000 - 1510$ (from part b) = $\$18490$ [1]

Printer C:

Use reducing balance depreciation equation:

$$V = P \times (1 - r/100)^t$$

$$\therefore V = 20000 \times (1 - \frac{1.55}{100})^5$$

$\therefore V = \$18{,}497.31$ [1]

Therefore, as Printer B's value is the lowest ($18,490) after 5 years, printer B has depreciated the most. [1]

Question 4
Tuesday's price =$2000 \times 1.34 = \$2680$

Wednesday's price = $2680 \times 0.84 = \$2251.2$

Percentage change = $\frac{amount\ of\ change}{original\ value} \times 100$

∴ amount of change = $2251.20 - 2000 = \$251.20$ [1]

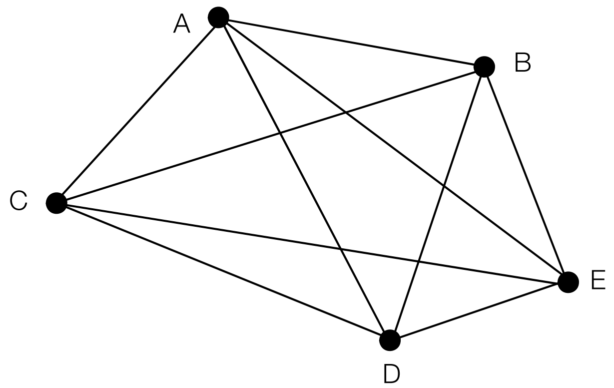
∴ Percentage change = $\frac{251.20}{2000} \times 100 = 12.56\%$ [1]

Question 5
$\$1{,}000{,}000$ [1]

Perpetuities only pay out the interest earned at the end of each period, and therefore the original investment remains the same forever.

Module 5: Networks and decision mathematics

Question 1a



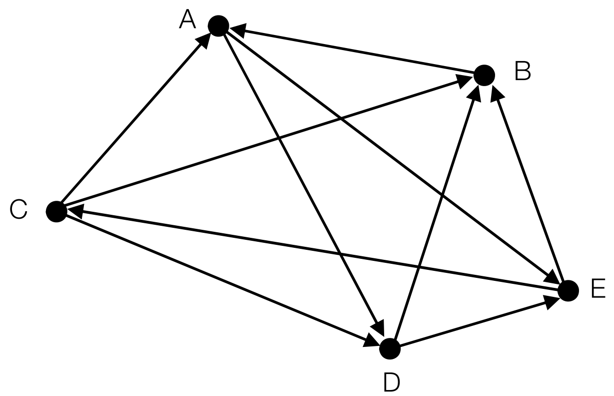
[1]

Question 1b

10 [1]

Count the number of edges. Alternatively use the formula, $\textbf{edges} = \frac{n(n-1)}{2}$, where n is the number of vertices in the graph.

Question 1c



[1]

Question 1d

$$M = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$ [2]

Question 2a

AFEDCB [1]

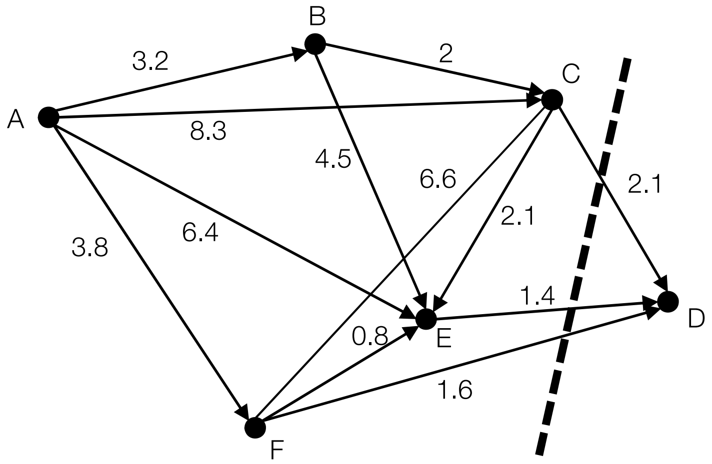
Question 2b

No, because there are more than two vertices in the graph which have an odd degree [1]

Euler path [1]

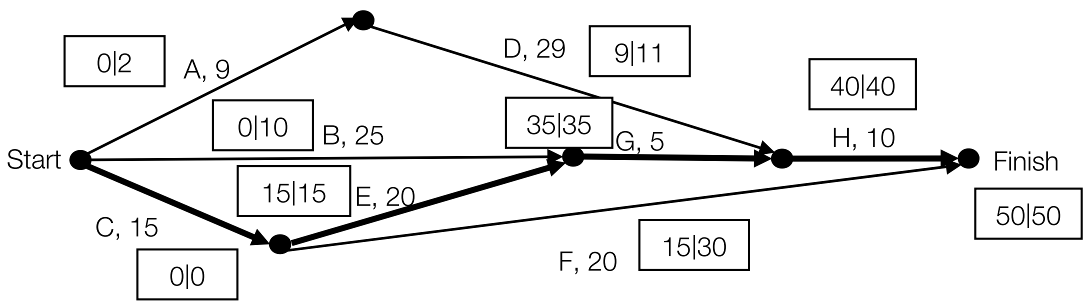
Question 2c
510 cars on average per hour. [1]

Find the minimum cut for the maximum flow. Hence 2.1 + 1.4 + 1.6 = 5.1 hundreds of cars per hour = 510 cars per hour.



Question 3a
The minimum time is 50 minutes. [1]
The critical path is CEGH. [1]

By completing forward scanning and backward scanning the EST and LST of the activities can be found.



Question 3b
2 minutes [1]
Slack (D)= LST-EST= 11-9=2 minutes

Question 3c
10 minutes [1]

Question 3d
Crash C, E, G or H by a maximum of 2 minutes.
The new completion time is 48 minutes. [1]

We should crash the activities on the critical path, ensuring that we do not overcrash so that other critical paths are created. Hence the maximum we can crash is 2 minutes as anything over 2 minutes will enable path ADH (with time = 48 minutes) to be the new critical path.

After reducing the completion time of C, E, G or H by 2 minutes, the new completion time will be 48 minutes.

Question 4
We need maximise the following matrix using the Hungarian Algorithm. So firstly, subtract the largest number from all the numbers in the matrix to find the maximum.

	W	X	Y	Z
A	170	240	420	210
B	250	180	190	200
C	290	140	310	220
D	110	200	170	140

	W	X	Y	Z
A	420-170	420- 240	420-420	420-210
B	420-250	420-180	420-190	420-200
C	420-290	420-140	420-310	420-220
D	420-110	420-200	420-170	420-140

	W	X	Y	Z
A	250	180	0	210
B	170	240	230	220
C	130	280	110	200
D	310	220	250	280

Now proceed with the Hungarian Algorithm as normal. First perform row reduction by subtracting the numbers in each row by the smallest number in that row.

	W	X	Y	Z
A	250	180	0	210
B	0	70	60	50
C	20	170	0	90
D	90	0	30	60

Now perform column reduction by subtracting the numbers in each column by the smallest number in that column.

	W	X	Y	Z
A	250	180	0	160
B	0	70	60	0
C	20	170	0	40
D	90	0	30	10

An allocation cannot yet be made, proceed forward with the Hungarian algorithm. Cover the zero elements with the minimum number of lines.

	W	X	Y	Z
A	250	180	0	160
B	0	70	60	0
C	20	170	0	40
D	90	0	30	10

Add the minimum uncovered number (20) to each of the rows and columns that are covered. Where lines intersect, add this number twice.

	W	X	Y	Z
A	250	180	20	160
B	20	90	80	20
C	20	170	20	40
D	110	20	50	30

Now subtract the minimum uncovered number (20) from **all** entries.

	W	X	Y	Z
A	230	160	0	140
B	0	70	60	0
C	0	150	0	20
D	90	0	30	10

[1]

An allocation can now be made so that each person receives a different job and all jobs are allocated.

Person A: Job Y
Person B: Job Z
Person C: Job W
Person D: Job X

The maximum profit is 290 + 200 + 420 + 200 = $1110 [1]

Module 6: Matrices

Question 1a

Paris, Amsterdam, Rome, Zagreb [1]

Question 1b

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$ [2]

Question 2a

$$\begin{bmatrix} 0.40 & 0.20 \\ 0.60 & 0.80 \end{bmatrix}$$ [1]

Question 2b

0.2 × 5 = 1

It is expected that one of the group will go [1]

Question 3a

$$\begin{bmatrix} 0.4 & 0.1 & 0.05 & 0.03 \\ 0.3 & 0.4 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.6 & 0.17 \\ 0.1 & 0.3 & .25 & 0.7 \end{bmatrix}$$

[3 for all rows correct, 0.5 marks off for each incorrect value]

Question 3b

3.00% [1]

Question 3c

$$\begin{bmatrix} 1000 \\ 3000 \\ 2000 \\ 500 \end{bmatrix}$$ [1]

Question 3d

$S_2 = T^2 \times S_0$

$$T = \begin{bmatrix} 0.4 & 0.1 & 0.05 & 0.03 \\ 0.3 & 0.4 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.6 & 0.17 \\ 0.1 & 0.3 & .25 & 0.7 \end{bmatrix}$$

$$S_0 = \begin{bmatrix} 1000 \\ 3000 \\ 2000 \\ 500 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 660.75 \\ 1338 \\ 2078.5 \\ 2422.75 \end{bmatrix}$$ [1]

Therefore, 1338 students receive a credit. [1]

Question 3e

$$\begin{bmatrix} 0 & 0.1 & 0.05 & 0.03 \\ 0 & 0.4 & 0.1 & 0.1 \\ 0 & 0.2 & 0.6 & 0.17 \\ 0 & 0.3 & .25 & 0.7 \end{bmatrix}$$ [1]

Question 3f

In semester 1 2013, 1000 students receive a pass. So 1000 students will drop out. [1]

Question 3g

Long term estimation = T^{50} x S_0

Majority of students will receive a HD [1]