B

Free Exam
for 2006-15 VCE study design

Engage Education Foundation

Units 3 and 4 Further Maths: Exam 2

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Core: Data analysis

Question 1a

Team	Minimum	1st Quartile	Median	3rd Quartile	Maximum
Wombats	4	11	14.5	18	42
Pandas	7	11	14.5	19	25

[1] for each correct 5 number summary

Question 1b

The Wombats have 3 outliers. The values are 32, 34, 42 [1]

Questions 2a

X is the independent variable, which is the ages of the spectators.

Y is the dependent variable, which is the amount of cash in their wallet.

$\text{amount of cash in spectator's wallet} = 2.6232 \times \text{age} + 9.2199$ [1]

Questions 2b

As Andrew is older he will have an extra $1.5 \times 2.632 \approx \$3.95$ more in his wallet. [1]

Questions 2c

$r^2 = 0.9132$ means that 91.32% [1] of the variation in the cash in the wallets of the spectators can be explained by the variation in the spectators' ages. [1]

Questions 3a

The graph has a clear seasonal trend. The sales are highest during winter, as seen by the peak on the time series. During summer and spring the sales are lowest, as evidenced by the trough. [1]

Question 3b

Year	2010	2011	2012
Average	7.775	7.863	7.600

[1]

Question 3c

Season	Summer	Autumn	Winter	Spring
Seasonal Index	0.701	1.084	1.507	0.708

[1]

Question 3d

$\text{time period } 11 = \text{winter } 2012 = \frac{11}{1.507} \approx 7.299$ [1]

Question 3e

$7.7722 \times 0.708 \approx 5.50$. This corresponds to time code 4, which is spring 2010. [1]

Question 4a

We can see an overall curved shape with the data. As the data is not linearly correlated a linear model may not necessarily be the model to choose. [1]

Question 4b

$y = x^2$ or $\text{percentage ill} = (\text{day number})^2$ is the most appropriate transformation given the curvature of the data. [1]

Question 4c

$\text{predicted value} = 1.7016 + 0.2025 \times (2^2) \approx 2.5516$

$\text{residual value} = \text{actual} - \text{predicted} = 2.44 - 2.5516 = -0.0716$ [1]

Modules

Module 1: Number patterns

Question 1a

30cm, 12cm, 4.8cm, 1.92cm,… [1]

Question 1b

$r = \frac{t_2}{t_1} = \frac{12}{30} = 0.4$

$r = \frac{t_3}{t_2} = \frac{4.8}{12} = 0.4$ [1]

Question 1c

Recognising that the change in height is in fact a geometric sequence with $a = 30cm$ and $r = 0.40$ [1]

Performing a sum to infinity $S_\infty = \frac{30}{1-0.40} = 50cm$. So in total, Alexander's height from the age of 10 will grow an extra 50cm. [1]

Alexander's eventual height will be $100cm + 50cm = 150cm$ [1]

Question 1d

Yes, from part c above. We see that Alexander will eventually grow to be 150cm. Unfortunately for him, Alexander will have to wait infinitely long before this occurs.

Question 2a

$5{,}000 \times 0.05 = 250 = d$. So each year, an extra $250 in interest is put into the account. [1]

The 7th term would be $t_7 = 5{,}000 + (7-1) \times 250 = 6{,}500$ [1]

Question 2b

$R = 1 + \frac{4.5}{100} = 1.045$ [1]

$t_7 = 5{,}000 \times 1.045^6 = \$6{,}511.30$ [1]

Question 2c

$R = 1 + \frac{6}{100} = 1.06$ [1]

$b = -90$ [1]. Note, that b must be negative, as we are subtracting this money away from the account balance.

Question 2d

$t_6 = 1.06 \times 5{,}918.67 - 90 = \$6{,}183.79$ [1]

$t_7 = 1.06 \times 6{,}183.79 - 90 = \$6{,}464.82$ [1]

Question 2e

Comparing the different bank plans. The 7^{th} term of the sequences correspond to the end of the 6^{th} year.

Bank A	\$6,500
Bank B	\$6,511.30
Bank C	\$6,464.82

Oscar should choose Bank B as it has the largest bank balance at the end of 6 years. [1]

Module 2: Geometry and Trigonometry

Question 1a

Application of the cosine rule:

$\text{run} = \sqrt{(2.5)^2 + (12.3)^2 - 2 \times 2.5 \times 12.3 \times \cos(62°)} \approx 11.34km$ [1]

Question 1b

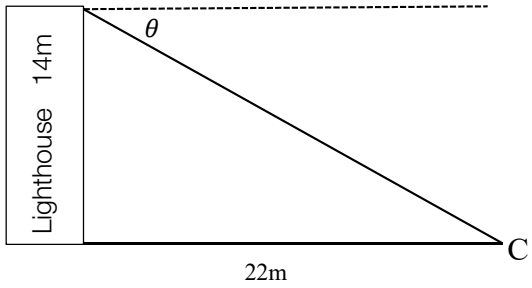
First calculate the angle made between the swim and run legs:

$\angle RSC = \sin^{-1}(\frac{12.3 \times \sin 62°}{11.34}) \approx 73.274°$ [1]

To find the true bearing that the spectators should turn take the supplement of this angle:

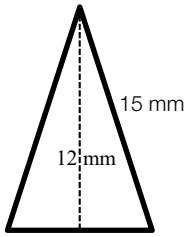
$180° - 73.274° = 106.73°T$ [1]

Question 1c



From the diagram the angle of depression is $\theta = \tan^{-1}(\frac{14m}{22m}) \approx 32.47°$ [1]

Question 2a



Using Pythagoras $\frac{base}{2} = \sqrt{15^2 - 12^2} = 9mm$

So the base length is 18mm [1]

Question 2b

$\text{area of 1 spike} = \frac{1}{2} \times 18 \times 12 = 108mm^2$

$\text{total area of silver} = 8 \times 48 = 864mm^2$ [1]

$\text{area of circle} = 864mm^2 = \pi * r^2$

$\text{radius} = \sqrt{\frac{864}{\pi}} \approx 16.58\, mm$ [1]

Question 2c

Area of each spike is now $\frac{864}{3} = 288mm^2$

If she wants to maintain the base length what should be the new height of the spike:

$288mm^2 = \frac{1}{2} \times 18 \times h$, Hence the new height would be $32mm$. [1]

if she wants to maintain the height what should be the new base length of the spike:

$288mm^2 = \frac{1}{2} \times b \times 12$, Hence the new base length would be 48mm [1]

Question 3a

Using a scale factor

$k^3 = 125$ hence $k = 5$ [1]
$r = 5 \times 3.385 = 16.926cm$

$h = 5 \times 10 = 50cm$[1]

Question 3b

$\text{Volume of smaller cone} = \frac{1}{3} \times \pi \times (3.385)^2 \times 10 \approx 120cm^3$ [1]

$\text{Volume of large cone} = 125 \times 120cm^3 = 15{,}000cm^3$ [1]

Question 3c

$\text{Total volume of trough} = 1000 \times \text{volume of large cone} = 15{,}000{,}000cm^3$

$\text{Total volume of trough} = 150cm \times 150cm \times \text{length} = 150{,}000{,}000cm^3$ [1]

$\text{length} = \frac{15{,}000{,}000cm^3}{150cm \times 150cm} \approx 666.67cm$ [1]

Students should convert the height and width of the trough from 1.5m to 150cm in order to make calculations easier.

Module 3: Graphs and Relations

Question 1a

$\text{Distance from point B} = 65 \times x \text{ hours}$ [1]

Bruce's house can be considered to be the origin, hence the y-axis will represent the distance travelled north of it.

Question 1b

$\text{Distance from point B} = -385km + 100 \times x \text{ hours}$ [1] for the correct gradient

[1] for correctly identifying the y-intercept to be -385. At the beginning of the trip, Sally is 385km south of Bruce. Hence with respect to Bruce's home, Sally has travelled -385km north.

Question 1c

$0 = -385km + 100x$

$x = \frac{385}{100} = 3.85\ hours$

$3.85\ hours\ \times 60 = 231\ minutes$

Question 1d

Solving $-385 + 100x = 65x$

$\frac{38535}{3503} = x = 11\ hours$ [1]

Question 1e

Sally will have travelled 1,100 km in total [1] This is regardless of her position with respect to Bruce's home. She has driven for 11 hours and 100km/hr

Question 2a

$x = 1000 \text{ liters of Xtreme Cola produced}$ [1]

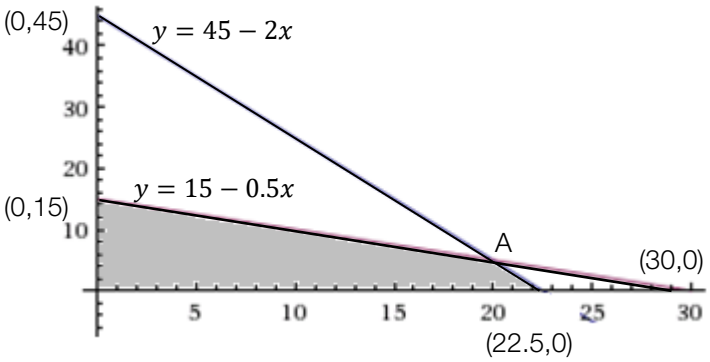
$y = 1000 \text{ liters of dYet Cola produced}$ [1]

Question 2b

$\text{Darren: } 20x + 10y \leq 450\ minutes$ [1]

$\text{William: } 10x + 20y \leq 300\ minutes$ [1]

Question 2c



[1] for stating that $(0,15) and\ (22.5,0)$ as a vertex of the feasible region.

[2] for correctly stating that vertex $A = (20,5)$ by solving the intersection of the linear equations.

[1] for correctly shading in the feasible region.

Question 2d

$\text{profit} = 150 \times x + 250 \times y$ [1]

Substituting all the vertices of the feasible region we find that (20,5) gives the maximum profit of $4,250 [1]

Module 4: Business-related mathematics

Question 1a

$\text{total amount} = 50 \times 12 \times 5$

$= \$3000$ [1]

Question 1b

$\text{total interest} = 3000 - 2500$

$= \$500$ [1]

Question 1c

$SI = \frac{Prt}{100}$

$500 = \frac{2500 \times r \times 5}{100}$

$r = 4\%$ [1]

Question 1d

$\text{cash price} = 2500 \times 1.25 \times 1.2$

$= \$3750$[1]

Question 2a

$A = P \times (1 + \frac{r \times {}^{1}/_{n}}{100})^{n \times t}$

$A = 5000 \times (1 + \frac{7.5 \times {}^{1}/_{12}}{100})^{12 \times 2}$ [1]

$A = \$5806.46$ [1]

Question 2b

When $t = 4, A = \$6743$

$\text{interest} = 6743 - 5000$

$= \$1743$ [1]

Question 3a

$\text{GST price} = \text{original price} \times 1.1$

$\therefore \text{original price} = 1445/1.1$

$= \$1313.64$ [1]

Question 3b

$\text{annual depreciation} = \frac{(1445 - 700)}{5}$

$= \$149$ [1]

Question 3c

$BV_5 = 1445 - (350 \times 5 \times 0.36)$

$= \$815$ [1]

Question 4a

$BV = P \times (1 - \frac{r \times {}^{1}/_{n}}{100})^{n \times t}$

$BV = 10000 \times (1 - \frac{12}{100})^{5}$

$= \$5277.32$ [1]

Question 4b

$4000 = 10000 \times (1 - \frac{r}{100})^{5}$ [1]

$r = 16.7\%$ [1]

Question 5

Use finance solver on calculator

$\text{Interest on loan A} = \$360410$

$\text{Interest on loan B} = \$182205$ [1]

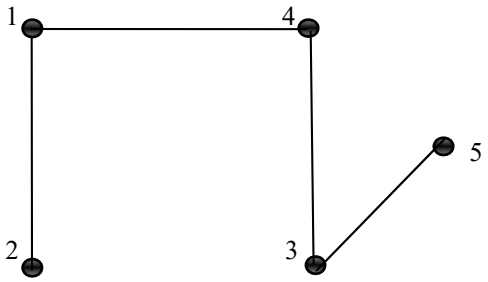
∴ Shannon should choose Bank B [1]

Module 5: Networks and decision mathematics

Question 1a

4 [1]

Question 1b



Answers will vary, but correct as long as there are 4 edges drawn so that each vertices is connected to at least one other. [1]

Question 2a

7.3km [1]

Question 2b

AGFEBCD [1]

Question 2c

Euler/Eularian path [1]

Question 2d

Between either towns A, B, C or E. [1] e.g. C and A

Question 3a

80 minutes [1]

Question 3b

Time taken to complete activities B, E, F and H is 55 minutes

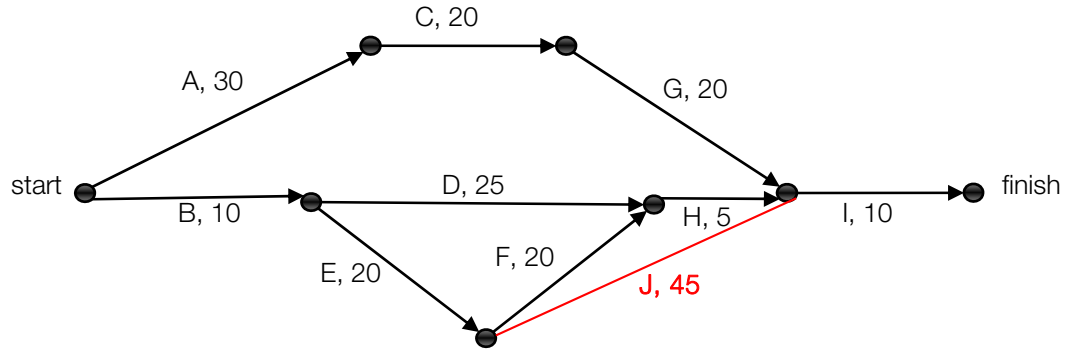
Time taken to complete activities A, C and G is 70 minutes [1]

∴slack time for H is 15 minutes [1]

Question 3c

70 minutes [1]

Question 3d

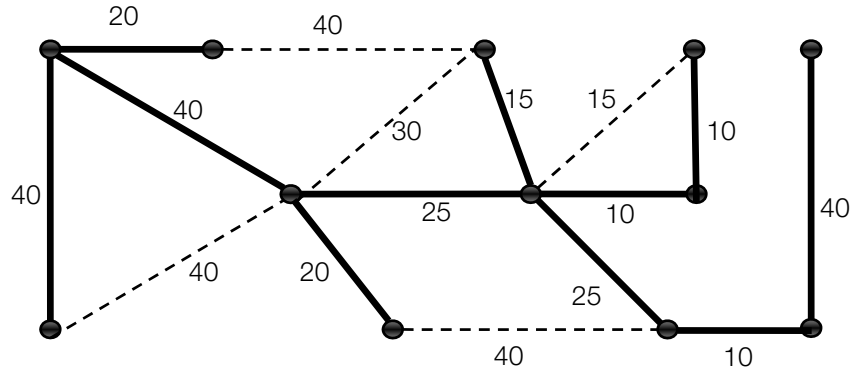


[1]

Question 3e

85 minutes [1]

Question 4a



Answers may vary [1]

Question 4b

255 metres [1]

Question 4c

$38250 [1]

Module 6: Matrices

Question 1a

$\begin{bmatrix} 60 & 150 \\ 45 & 70 \end{bmatrix}$ [1]

Question 1b

$AB = [1120]$ [1]

Question 1c

$\text{order of BA} = 4 \times 4$ [1]

Question 1d

The product of A and B gives the total amount spent by the customer in the boutique. [1]

Question 1e

$$[60 \;\; 45 \;\; 18 \;\; 25] \times \begin{bmatrix} 150 \\ 70 \\ 200 \\ 80 \end{bmatrix}$$ [1]

$\text{amount earned} = \$17750$ [1]

Question 2a

$A_2 = \begin{bmatrix} 2550 \\ 1550 \end{bmatrix}$ [1]

Question 2b

4100 caps [1]

Question 2c

$A_{10} = \begin{bmatrix} 2806 \\ 1806 \end{bmatrix}$ [1]

Question 2d

The number of people attending Dragons games is expected to increase and then stay steady at a value of approximately 3000 supporters. [1]

Question 2e

The number of people attending Dragons games is expected to decrease and then stay steady at a value of approximately 600 supporters. [1]

Question 3a

117 students [1]

Question 3b

176 students attend in week 4 and 188 students attend in week 3. [1]

∴The number of students expected to not return for extra lessons is 12. [1]

Question 3c

Over time the expected number of students attending extra lessons steadily decreases. [1]