A

Free Exam
for 2006-15 VCE study design

Engage Education Foundation

Units 3 and 4 Further Maths: Exam 2

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Core: Data analysis

Question 1a
55 [1]

Question 1b
65 – 45 = 20 [1]

Question 1c
No they don't because the median score doesn't increase with the year level. Year nine's median score is 55, year ten's median score is 80 and year eleven's median score is 55. So they are not positively related. [1]

Question 1d
No. [1]

1.5 x IQR = 1.5 x (65-50) = 22.5
The upper fence is therefore at Q_3 + 22.5 = 65 + 22.5 = 87.5

The score of 82 is within the upper fence and therefore is not an outlier. [1]

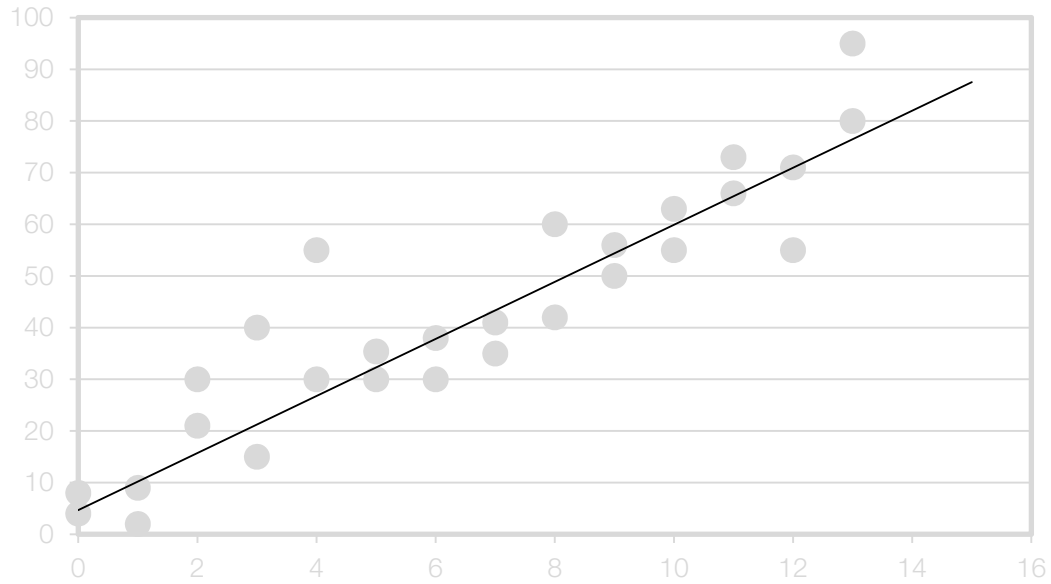
Question 2a
$\frac{5}{12} = 41.7\%$ [1]

Question 2b
Mean = 47.7 [1]

Median = 49 [1]

Question 2c
This is because the mean percentage of boys in each class is 47% which would mean that on average there are less than 50% boys in every class and therefore there has to be more than 50% girls in every class. This then means that there has to be more than 50% girls in the grade. [1]

Question 3a



[1 for regression line]

Question 3b

5.5 [1]

Question 3c

Grade = 4.7 + (5.5 x 15) = 87.2

Residual value = actual value – residual value = 94 – 87.2 = 6.8 [1]

Question 3d

Study hours per week [1]

Question 4a

Negative skewedness [1]

Question 4b

$\frac{5}{12} = 41.7\%$ [1]

Modules

Module 1: Number patterns

Question 1a

$P_n = 1398 - 45 \times n$ [1]

Question 1b

$P_6 = 1398 - (45 \times 6) = 1128$ [1]

Question 1c i

15 [1]

$723 = 1398 - (45 \times n)$
$n = 15$

Question 1c ii

31 [1]

$0 = 1398 - (45 \times n)$
$n = 31$

Question 1d

585 [1]

$n = 2$: $P_2 = 1308$
$n = 15$: $P_{15} = 723$

So $1308 - 723 = 585$

Question 2a

7091 [1]

Since $P_2 = 1.12 (P_1) + 45$,
$8990 = 1.12 (P_1) + 45$
$P_1 = 7987$

So $P_1 = 1.12 (P_0) + 45$
$7987 = 1.12 (P_0) + 45$
$P_0 = 7091$

Question 2b

1259 [1]

$P_3 = 1.12 (P_2) + 45 = 1.12 (8990) + 45 = 10114$
$P_4 = 1.12 (P_3) + 45 = 1.12 (10114) + 45 = 11373$
So $P_4 - P_3 = 11373 - 10114 = 1259$

Question 2c

12.0% [1]

Question 2d
7 years [1]

Use the sequence function on a graphics calculator to generate the following table:

2	8990.00
3	10113.80
4	11372.46
5	12782.15
6	14361.01
7	16129.33

Question 2e
$P_3 - P_2 = 10114 - 8990 = 1124$
$P_2 - P_1 = 8990 - 7987 = 1003$
Since $1124 \neq 1003$ there is no common difference and hence the sequence is not arithmetic [1]

Question 3a
18, since 156 – 138 = 18 and 138 – 120 = 18 [1]

Question 3b
192 [1]

Question 3c
S = 1 [1]
K = 18 [1]

Question 3d
9^{th} year [1]

Use the sequence function on a graphics calculator to generate the following table:

4	174.00
5	192.00
6	210.00
7	228.00
8	246.00
9	264.00

Module 2: Geometry and trigonometry

Question 1a

$\tan(28°) = \frac{x}{13.4}$

$x = 7.1$ m [1]

Question 1b

$\tan(\theta) = \frac{7.12}{10.4}$

$\theta = 34.4°$ [1]

Question 1c

$30\text{cm} = 7.12$ m

$30 \times 10^{-2} : 7.12$

$\therefore scale\ factor = \frac{1}{24}$ [1]

Question 1d

Need to find angle BAC = 180 – 80 – 23 = 77°

Using cosine rule CB = 35.5 m [1]

Question 1e

Need to find angle CBA.

Use sine rule, therefore CBA = $\sin^{-1}\frac{36\sin 77}{35.5}$ = 81.4° [1]

Therefore the bearing of C from B is 360 – (81.4-23) = 302°T [1]

Question 2a

Let X be the point directly below O at the same height as AB. Then AX = $\sqrt{5^2 + 3^2} = \sqrt{34}$.

Using Pythagoras' theorem a second time, OX= $\sqrt{6.3^2 - \left(\sqrt{34}\right)^2} = \sqrt{5.69} = 2.4$ m

Therefore the height of the pavilion is $2.4 + 3 = 5.4$ m [1]

Question 2b

TSA of the roof = $4 \times \left(0.5 \times 10 \times \sqrt{6.3^2 - 5^2}\right) = 76.7\ m^2$ [1]

Question 2c

$V = \frac{1}{3}Ah + Ah = 227.71\ m^3$ [1]

Question 2d

Scale factor is $\left(\frac{1}{2}\right)^2$ therefore the reduction in volume is 25% [1]

Question 3a

Converting from mm to cm, divide by 10 as 1 cm = 10 mm.

$V = \pi r^2 h + \frac{1}{2} \times \frac{4}{3}\pi r^3$ = 3845.3 cm^3 [1]

Question 3b

$\text{TSA} = 2\pi rh + \pi r^2 + \frac{4\pi r^2}{2}$ = 1470.3 cm^2 [1]

Question 3c

This question requires the application of Pythagoras theorem. The largest diagonal is the diagonal that joins a top corner to its opposite bottom corner.

$\text{bottom diagonal} = \sqrt{30^2 + 35^2} = 46.0977$ [1]

$\text{largest diagonal} = \sqrt{20^2 + 46.0977^2} = 50.25 = 50 \text{ cm}$ [1]

Module 3: Graphs and relations

Question 1a
$2.00 [1]

Question 1b
$6.00 [1]

Question 1c
$\text{Charge} = 5 \times (\text{Weight} + 1)$
Therefore, $5 \times (6 + 1) = \$35$ [2]

Question 1d i
$3b + 2p = 10.20$ [1]
$2b + 3p = 9.30$ [1]

Question 1d ii
Solve to find that:
$b = 2.4$ [1]
$p = 1.5$ [1]
Therefore, $2.4 + 1.5 = \$3.90$ [1]

Question 2a
sub in $n = 2$ in equation $G = kt^n$ [1]
sub in any corresponding values
eg. $1 = k2^2$
$4k = 1$
$k = \frac{1}{4}$ [1]

Question 2b
3 years = 36 months
Hence $\frac{1}{4}(36^2) = 324$ [2]

Question 2c
Revenue = Cost
Hence, $16281 \div 324 = \$50.25$ [2]

Module 4: Business-related mathematics

Question 1a

$$I = P\left(1 + \frac{r/n}{100}\right)^{nt} - P$$

$$I = 399\left(1 + \frac{15/12}{100}\right)^{12\times\frac{3}{12}} - 399$$

$I = \$15.15$ [1]

Question 1b

Use the compound interest formula to determine the amount of interest per month: $I = P\left(1 + \frac{r/n}{100}\right)^{nt} - P$

The balance of the loan at the end of each month will be: $Balance = P + I - 140$ [1]

End of month	Interest ($)	Repayment ($)	Balance of loan ($)
1	4.99	140	263.99
2	3.30	140	127.29
3	1.59	128.88	0

Thus is will take 3 months to pay off the coffee machine. [1]

Question 2a

$\text{Total paid} = 50 + 8 \times 45 = 410$

$\text{Interest} = 410 - 399 = 11$

Therefore Joe pays $11 in interest. [1]

Question 2b

$r_f = \frac{100I}{Pt} = \frac{100\times11}{(399-50)\times\frac{8}{12}} = 4.73\%\text{ per annum}$ [1]

A common mistake would be to forget to minus the deposit from the price of the coffee machine. As this $349 is the amount owing after the deposit is paid.

Question 2c

$r_e = \frac{100I}{Pt} \times \frac{2n}{n+1} = \frac{100\times11}{(399-50)\times\frac{8}{12}} \times \frac{2\times8}{8+1} = 8.40\%\text{ per annum}$ [1]

Question 3a

$$A = P\left(1 + \frac{r/n}{100}\right)^{nt}$$

$$A = 1000\left(1 + \frac{\frac{8.3}{4}}{100}\right)^{4} = 1085.619$$

The value of her investment after one year would be $1085.62 [1]

Question 3b

$$A = P\left(1 + \frac{r/n}{100}\right)^{nt}$$

$$3000 = 1000\left(1 + \frac{\frac{8.3}{4}}{100}\right)^{4t}$$

Using a calculator to solve for t we get t=13.3731 years.

Thus to the nearest year Kelly would have to keep her money invested for 14 years [1]

A common mistake would be to state 13 years, however at 13 years she will still not have $3000 but slightly less.

Question 3c

This is equivalent of investing $3000 at 2.1% interest compounding annually.

$A = P\left(1 + \frac{r}{100}\right)^{t}$ [1] (One mark for recognizing this is an application of compound interest)

$$A = 3000\left(1 + \frac{2.1}{100}\right)^{14}$$

$$A = \$4013.12$$

The price of the coffee machine would rise to $4013.12 [1].

If the value for t used was 13 and the answer $3930.57 was obtained the full mark is also awarded.

Question 4a

$N = 12$
$I = 8.2$
$PV = 2000$
$PMT = -174.16$
$FV = 0$
$P/Y = 12$

Therefore Kelly must make monthly repayments of $174.16 [1]

Question 4b

$N = 89.28$
$I = 8.2$
$PV = 2000$
$PMT = -30$
$FV = 0$
$P/Y = 12$

Then must round N down and then use the TMV solver to find FV at this value for N which will correspond to the value of the last repayment

$N = 89$
$I = 8.2$
$PV = 2000$
$PMT = -30$
$FV = -8.29$
$P/Y = 12$

Therefore will take 90 months to pay off the loan and the last repayment will be \$8.29 [2]

Question 4c

$\text{Interest paid} = 30 \times N + (\text{answer to question 4b}) - 2000 = \$678.29$ [1]

Where N has been rounded down to the nearest integer, as it is not realistic to have a fractional number of repayments

Module 5: Networks and decision mathematics

Question 1a

6 [1]

Do not forget the infinite face.

Question 1b

$\deg A = 3, \deg B = 4, \deg C = 3$

Sum of all degrees of the vertices = 3 + 4 + 3 = 10 [1]

Question 1c

	A	B	C
A	1	2	1
B	2	1	1
C	1	1	1

[1 for matrix]

Question 1d

It means that Y is not reachable in one step from either W, X or Z. [1]

Question 2a

It is not planar because the graph cannot be drawn in such a way that no two edges meet (or have common points). [1]

Question 2b

B-C-A-B-D-A-E-D-C, or any other path that includes every edge just once. [1]

Question 2c

A-B-C-D-E-A, or any other circuit that passes through every vertex only once. Note the question asks for a circuit therefore must begin and end at the same node. [1]

Question 2d

Only count the edges that contribute to the flow of the network, as this is not specified two answers are possible.

Cut 1 = 4 + 7 + 6 = 17 [1]

Or

Cut 1 = 5 + 5 = 10 [1]

Question 2e

Maximum flow of tourist = capacity of minimum cut that separates A from C [1]. Mark can also be gained by an appropriate sketch showing cuts that separate vertices A and C from each other.

Minimum cut is 8; therefore the maximum number of tourists that can travel from A to C is 8 people. [1]

Question 3a

52, 3, 65, 0 [1]

Question 3b

Supplying to	Factory based at			
	Location 1	Location 2	Location 3	Location 4
Shop A	5	13	10	0
Shop B	0	0	15	0
Shop C	22	1	15	0
Shop D	5	3	0	0

Question 3c

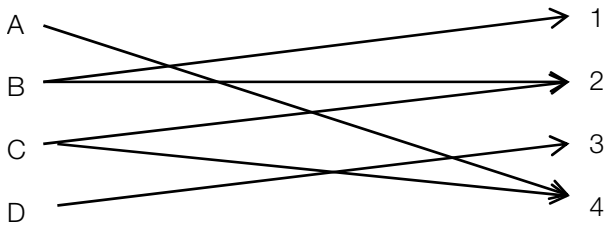
The number of lines that cover the zeros in the table from question 3b does not equal the number of rows, therefore must now add the minimum uncovered element (that is 1) to the elements that are covered.

Supplying to	Factory based at			
	Location 1	Location 2	Location 3	Location 4
Shop A	5	13	10	1
Shop B	1	1	16	2
Shop C	22	1	15	1
Shop D	6	4	1	2

Now subtract the minimum uncovered number from all entries

Supplying to	Factory based at			
	Location 1	Location 2	Location 3	Location 4
Shop A	4	12	9	0
Shop B	0	0	15	1
Shop C	21	0	14	0
Shop D	5	3	0	1

Now the number of lines needed to cover the zeros equals the number of rows therefore we can now make our bipartite graph. The 0's indicate the required allocations.



Bipartite graph or similar reasoning [1]

Therefore,

- shop A must be supplied from factory at location 4
- shop B must be supplied from factory at location 1
- shop C must be supplied from factory at location 2
- shop D must be supplied from factory at location 3

[1 mark for answer]

Question 3d

Min cost is therefore 80 + 105 + 78 + 135 = $398

Module 6: Matrices

Question 1a

2 x 2 [1]

Question 1b

determinant $= ad - bc$
$= (3 \times 6) - (4 \times 5)$
$= -2$ [2]

Question 1c

$[1 \quad 2] \times \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} = [13 \quad 16]$ [1]

Question 2a

$\begin{bmatrix} 4 & 3 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 18 \end{bmatrix}$ [2]

Question 2b

$A^{-1} = \begin{bmatrix} -1/2 & -3/2 \\ 1 & 2 \end{bmatrix}$ [2]

Question 2c

$X = A^{-1} \times C$
$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/2 & -3/2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 12 \\ 18 \end{bmatrix}$
Solve. Therefore, $x = -33$ [1]
$y = 48$ [1]

Question 3a

$\begin{bmatrix} .4 & .75 \\ .6 & .25 \end{bmatrix}$ [1]

Question 3b

$\begin{bmatrix} .4 & .75 \\ .6 & .25 \end{bmatrix} \begin{bmatrix} 120 \\ 80 \end{bmatrix} = \begin{bmatrix} 108 \\ 92 \end{bmatrix}$ [1]
92 members will not play [1]

Question 3c

$\begin{bmatrix} .4 & .75 \\ .6 & .25 \end{bmatrix}^{50} \begin{bmatrix} 120 \\ 80 \end{bmatrix} = \begin{bmatrix} 111.111 \\ 88.8889 \end{bmatrix}$

Therefore, in the long term 111 will play [1] and 89 will not [1]