Mathematical Methods

Outcome 6- Probability

Two-state Markov Chains: the result of each trial is conditional on the result of the previous trial

$S_n = T \times S_{n-1} = T^n \times S_0$

The general structure of a transition matrix:

$$\begin{matrix} & A & B \\ A & \Pr(A|A) & \Pr(A|B) \\ B & \Pr(B|A) & \Pr(B|B) \end{matrix}$$

Steady state of a Markov chain:

$$\Pr(X_n = 0) = \frac{b}{a+b} \qquad \Pr(X_n = 1) = \frac{a}{a+b}$$

Geometric Distribution:

It is different to binomial and markov chain.

Typically a question will ask:

e.g. What is the probability that you have x failures before success:

$\Pr(X = x) = (1-p)^x \times p$

Continuous probability: it can take any value in a given interval

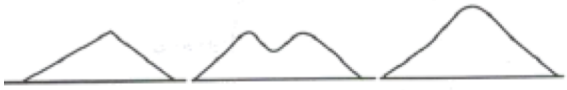
$$\Pr(a < X < b) = \Pr(a \leq X \leq b) = \int_a^b f(x)dx = F(b) - F(a) \text{ where } F'(x) = f(x)$$

The properties of the probability density function:

1. f(x)≥0 for all x
2. $\int_{-\infty}^{\infty} f(x)dx = 1$

Tip:

1. f(x) does not have to be smooth curve. They can be other shapes too!



Expected value of a continuous probability density function:

$$\mu = E(X) = \int_{-\infty}^{\infty} xf(x)dx$$

$$E[g(x)] = \int_{-\infty}^{\infty} g(x)f(x)dx$$

Variance

$$Var(X) = \sigma^2 = E(X^2) - [E(X)]^2 = \int x^2 f(x)dx - \mu^2$$

Percentile and the median

The median for continuous random variable is such that:

$\int_0^m f(x)dx = 0.5$

We can solve this on the calculator (CAS):

$solve\left(\int_{-\infty}^{m} f(x)dx = 0.5, m\right)$

The interquartile range of X is:

IQR= b-a

$$\int_{-\infty}^{a} f(x)dx = 0.25 \quad and \quad \int_{-\infty}^{b} f(x)dx = 0.75$$

Calculator tips:

We can determine normal probabilities using this

1. Main → Interactive → Distribution → normCDf/normPDf →enter in the values
2. Statistics → Calc → Distribution →Normal PD/Norm CD → enter in the values
3. Main → Type in normCDF(lower number, upper number, σ, μ)

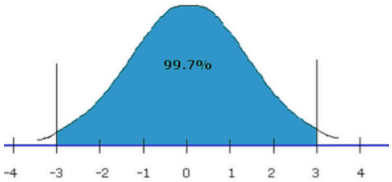
Normal Distribution: it may take any value within a given range, and no distinct, countable values.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad x \in R$$

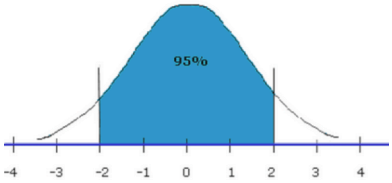
It has the following properties:

1. Symmetrical
2. Bell shaped
3. Mean=mode=median

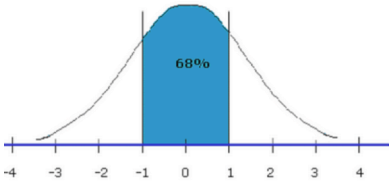
Standardisation and the 68-95-99.7% rule



99.7% of the values lie within three standard deviation of the means



95% of the values lie within two standard deviation of the mean



68% of the values lie within one standard deviation of the mean

To standardise, we use $Z=\frac{x-\mu}{\sigma}$