

Mathematical Methods

Outcome 5- Probability

Independent event: When the event is independent, whether or not another event (B) has occurred, it has no effect on the probability of A occurring

$$\Pr(A|B) = \Pr(A) \text{ and}$$

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

Mutually exclusive

$$\Pr(A \cap B) = 0$$

Conditional probability

It is the probability of an event (A) occurring when it is known that another event has occurred (B). It is usually written as $\Pr(A|B)$.

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Random variables: There are two types of random variables which is determined by the outcome of a random experiment:

1. Discrete random variable: can only take particular (discrete) values
2. Continuous random variables: can take any value in a given interval

Rules of the discrete probability functions:

1. $0 \leq p(x) \leq 1$ for all x

2. $\sum_x p(x) = 1$

3. $\Pr(a \leq X \leq b) = \sum_{x=a}^{x=b} p(x)$

Measures of centre

These are used to measure the centre of a discrete random variable

1. Expected value (mean value): used to determine the true mean value of a random variable from the probability distribution

$$E(X) = \sum_x x \cdot \Pr(X = x)$$

$$= \sum_x x \cdot p(x)$$

2. Mode: the most probable value of the random variable

$$E(aX + b) = aE(X) + b \text{ (a, b constant)}$$

3. Median: The middle value of the distribution (in which 50% of the distribution is large than it and 50% is less than it)

Tip:

1. It is important to note that the expected value of a function of X is not equal to that function of the expected value of X

Binomial distribution:

If x is the number of successes in n trials, with probability of each success equal to p, then:

$$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \quad x = 0, 1, \dots, n$$

$$\text{where } \binom{n}{x} = \frac{n!}{x!(n-x)!}$$

The sum of the binomial probabilities must equal to 1

Measure of variability

These are used to measure how close the values are to the mean and how spread out the probability distribution is.

1. Variance: measure the spread of the probability distribution from its expected value

$$\text{Var}(X) = E[(X - \mu)^2] \text{ OR } \text{Var}(X) = E(X)^2 - [E(X)]^2$$

2. Standard deviation: measures the spread of an unit

$$\text{sd}(X) = \sqrt{\text{Var}(X)}$$

Interpretation of standard deviation:

In general, for many random variables X:

$$\Pr(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx 0.95$$

Expected value of the binomial distribution:

$$E(X) = np$$

Variance and Standard deviation:

$$\sigma^2 = np(1-p)$$

$$\sigma = \sqrt{np(1-p)}$$

Calculator tip:

1. Plot binomial distributions

Casino Classpad 300 Calculator

To plot the graph:

1. Statistics
2. Calc
3. Distribution
4. Binomial PD
5. Enter in the values
6. Tap the graph button on top