Mathematical Methods

Outcome 4- Anti-derivative of functions

Integration (Anti-derivatives): is the reverse of differentiation

Rules for anti-differentiation

1. $\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + c$ $n \neq 1$
2. $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)}$ $n \neq 1$, $(ax+b)$ is a linear expression
3. $\int e^{kx}\,dx = \frac{1}{k}e^{kx} + c$

Circular functions:

$$\int \cos(kx)\,dx = \frac{1}{k}\sin(kx) + c$$

$$\int \sin(kx)\,dx = -\frac{1}{k}\cos(kx) + c$$

$$\int \sec^2(kx)\,dx = \frac{1}{k}\tan(kx) + c$$

Logarithmic functions:

$$\int \frac{1}{x}\,dx = \log_e|x| + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \log_e|f(x)| + c$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\log_e|ax+b| + c$$

The fundamental theorem of calculus

The exact area under a curve between x=a and x=b is given by:

$$Area = \int_a^b f(x)\,dx$$

$$= [F(x)]_a^b \quad where\ F'(x) = f(x)$$

$$= F(b) - F(a)$$

Tips:

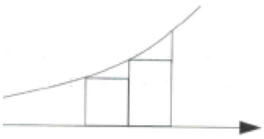
1. Note that $\int_a^b f(x)dx = -\int_b^a f(x)dx$

Anti-differentiation by recognition

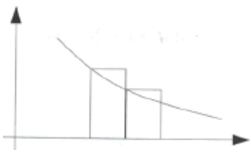
$\frac{d}{dx}[f(x)] = f'(x) = g(x),$ implies that $\int g(x)\,dx = f(x) + c$

Approximating area under curve by left rectangles

1. The rectangles are under the curve for an increasing function

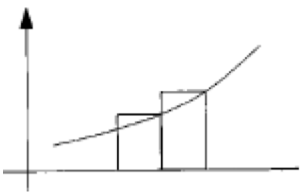


2. The rectangles are above the curve for a decreasing function



Approximating area under curve by right rectangles

1. The rectangles are above the curve for an increasing function



2. The rectangles are under the curve for a decreasing function

Properties of anti-derivatives and infinite integrals

1. $\int af(x) \pm bg(x)\,dx = a\int f(x)\,dx + b\int g(x)\,dx$

This means that the common factors from the expression can be remove before we anti-differentiate

2. $\int_a^b f(x)dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

This means that if b is the upper terminal of the first integral, as well as the lower terminal of the second integral, we can do the calculation together. (instead of doing it seperately)

3. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

By reversing the terminal, the sign needs to be changed

4. $\int_a^a f(x)dx = 0$

There is no area under the curve

5. $\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$