Mathematical Methods

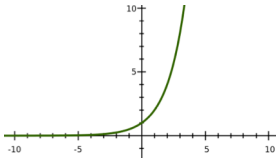
Outcome 2- Graphs & functions

Exponential function:

General exponential functions are of the form:

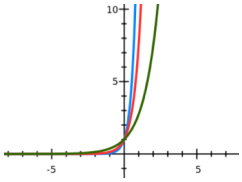
f:R->R, $f(x)= a^x$, where a is a positive real number ≠ 1

1. There is one asymptote (the x-axis)
2. All graphs pass through (0,1)



$f(x)= 2^x$

f:R->R, $f(x)= e^x$, where "e" is called Eulers's number. It is an irrational number and is exactly written as e.

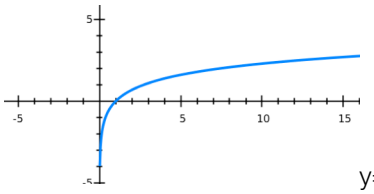


$f(x)= e^x$, $f(x)= 2e^x$, $f(x)= 3e^x$

Logarithmic function:

y= ln(x) or $y=\log_{10}(x)$

1. There is one asymptote (the y-axis)
2. All graphs pass through (1,0)
3. If the base is less than 1, for example $y=\log_a\frac{1}{2}$, this can be written as $y=-\log_a 2$



y=ln(x)

Tips:

1. Logarithmic and exponential functions are inverses of each other
2. Remember your index laws and log laws!

Circular functions:

sinθ is positive in the 1st and 2nd quadrant

cosθ is positive in the 1st and 4th quadrant

tanθ is positive in the 1st and 3rd quadrant

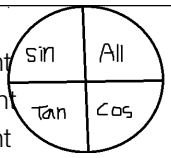
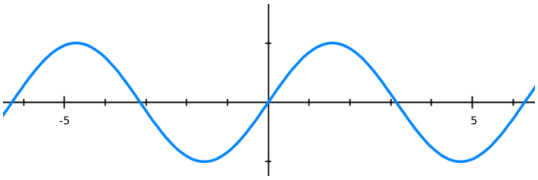


Table of exact values

Θ(θ°)	sinθ	cosθ	tanθ
0	0	0	0
$\frac{\pi}{6}$(30°)	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$(45°)	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$(60°)	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$(90°)	1	0	Undefined
π(180°)	0	1	0

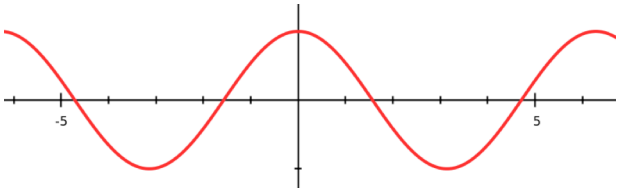
Sine graphs

y=asin(nx); period=$\frac{2\pi}{|n|}$; amplitude=$|a|$; range:[-a,a]



Cosine graphs

y=acos(nx); period= $\frac{2\pi}{|n|}$; amplitude= $|a|$; range [-a,a]

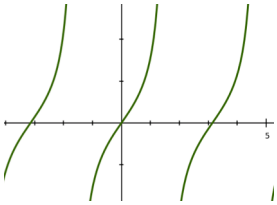


Tanθ graphs

y=atan(nx); period= $\frac{\pi}{|n|}$; range: R; asymptote:

x=$\frac{(2k+1)\pi}{2n}$

x-intercept: $\frac{k\pi}{n}$



Tips:

1. The amplitude of the graph is always positive, as it is distance (it is half the distance from the minimum to the maximum)
2. Memories the table of exact values and it will become handy before SACs/exams
3. Know your general solutions formula

Modulus function: also known as absolute value function

Expressed as $f(x)= |x|$

e.g. $y=|x|$,

$$y(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$