Further Mathematics

Matrices

Elements of a matrix

$$\begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} & x_{1,4} \\ x_{2,1} & x_{2,2} & x_{2,3} & x_{2,4} \\ x_{3,1} & x_{3,2} & x_{3,3} & x_{3,4} \end{bmatrix}$$

column 2 row 3

Order of a matrix

Column x row

When multiplying two matrices, the order of the product matrix has an order equal to the rows of the first matrix, and the columns of the second matrix.

Simultaneous equations

$$2x + 4y = 4$$
$$7x + 3y = 9$$

is written as

$$\begin{bmatrix} 2 & 4 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

Identity matrices

Identity matrices are square and have ones along the leading diagonal and the zeros elsewhere.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Determinant of a matrix

From a matrix A,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The determinant is given by det(A)=a*d-b*c

det = 0 means that there are indefinitely many solutions (same line) or there are no solutions (parallel lines with different y-intercepts).

Inverse matrix, A^{-1}

$$AA^{-1} = A^{-1}A = I$$

The inverse matrix is given by

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Note that ad – bc is equal to the determinant of matrix A

Addition and subtraction of matrices

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 5 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 4 & -1 \\ 3 & -6 & 5 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 8 & 8 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A - B = \begin{bmatrix} -4 & 0 & 4 \\ -2 & 11 & -6 \end{bmatrix}$$

Multiplying matrices by a scalar

$$C = 3\begin{bmatrix} 8 & 8 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 24 & 24 & 6 \\ 12 & -3 & 12 \end{bmatrix}$$

Multiplying matrices with matrices

- ensure the number of columns in the first matrix equals the number of rows in the second matrix
- use calculator

Solving simultaneous equations

$A^{-1}AX = A^{-1}C$
$IX = A^{-1}C$
$X = A^{-1}C$